## Compiling Techniques

Samson Abramsky
samson@dcs

## Why study compilers?

- To learn how to use them well.
- To learn how to write them.
- To illuminate programming language design.
- As an example of a large software system.
- To motivate interest in formal language theory.


## Course bias

- Not a theory course.
- Not a hardware/assembler course.
- Not a superficial survey of techniques.
- Concentrate on important ideas.
- Examine the ideas in implementations.


## Course text

- There are two editions of the course text; and there are three versions of each of them!
- You can choose either
- "Modern Compiler Implementation", by Andrew Appel, Cambridge University Press, 1998.
Price $£ 27.95$.
- or "Modern Compiler Implementation: Basic Techniques", by Andrew Appel, Cambridge University Press, 1997. Price £19.95.
- There are versions for the languages C, ML and Java.
- We study Part One of the book. This is common to both editions.


## Coursework

- Choose an implementation language (C, ML, Java, C++, ...)
- Obtain the (partial) source code and the Tiger'99 Reference Manual from the CS3 Compiling Techniques Web page.
- Working in groups or otherwise, develop a compiler for Tiger'99 as described in the reference manual.
- Groups can use the Compiling Techniques group accounts (with the user ids ct00, ct01, ct02, ...). Mail support@dcs telling them the names of the people in your group.
- Submit source, SUN Solaris executable and documentation (a README file)
- Deadline: End of Week 9 of this term.


## Tiger and Tiger'99

- Tiger ' 99 is a dialect of the Tiger language described in Andrew Appel's textbook "Modern Compiler Implementation".
- Although Tiger '99 has many features in common with Tiger it adds some new syntax and concepts while taking away others. Thus it is neither a subset nor a superset of Tiger
- However, the skills which are needed to know how to compile the language are those which can be learned from careful study of Appel's textbook.

2. Lexical analysis, LEX; basic parsing.
\{if (!strncmp (s, "0.0", 3))
3. Predictive parsing; concepts of LR parsing
4. YACC; Abstract syntax, semantic actions, parse trees.
5. Semantic analysis, tables, environments, type-checking.
6. Activation records, stack frames, variables escaping.
7. Intermediate representations, basic blocks and traces.
8. Instruction selection, tree patterns and tiling.
9. Liveness analysis, control flow and data flow.

## Lexical analysis

- The first phase of compilation
- White space and comments are removed.
- The input is converted into a sequence of lexical tokens.
- A token can then be treated as a unit of the grammar.


## Lexical tokens

Examples:
foo (ID), 73 (INT), 66.1 (REAL), if (IF), ! = (NEQ), ( (LPAREN), ) (RPAREN)
Non-examples:
/* huh? */ (comment),
\#define NUM 5 (preprocessor directive), NUM (macro)
where there are zero or more symbols on the right-hand side. Each symbol is either terminal, meaning that it is a token from the alphabet of strings in the language, or non-terminal, meaning that it appears on the left-hand side of a production. No terminal symbol can ever appear on the left-hand side of a production and there is only one non-terminal there (together these justify the name context-free). Finally, one of the productions is distinguished as the start symbol of the grammar.

## A grammar for straight-line programs

| 1. $S \rightarrow S ; S$ | (compound statements) |
| :--- | ---: |
| 2. $S \rightarrow \mathrm{id}:=E$ | (assignment statements) |
| 3. $S \rightarrow \operatorname{print}(L)$ | (print statements) |
| 4. $E \rightarrow \mathrm{id}$ | (identifier usage) |
| 5. $E \rightarrow$ num | (numerical values) |
| 6. $E \rightarrow E+E$ | (addition) |
| 7. $E \rightarrow(S, E)$ | (comma expressions) |
| 8. $L \rightarrow E$ | (singleton lists) |
| 9. $L \rightarrow L, E$ | (non-singleton lists) |

1. $S \rightarrow S ; S$
2. $S \rightarrow$ in $: E$
3. $E \rightarrow \mathrm{id}$
4. $E \rightarrow$ num
$E \rightarrow E+E$
$L \rightarrow E$
5. $L \rightarrow L, E$
(compound statements)
(assignment statements)
(print statements)
(identifier usage)
(addition)
(singleton lists)
(non-singleton lists)

Examples: $\mathrm{a}:=7$; b := c + (d := $5+6, \mathrm{~d})$
Non-examples: a := 7 ; b := (d, d) print ()

## A derivation

$$
\begin{aligned}
& \underline{S} \\
& S ; \underline{S} \\
& \underline{S} ; \mathrm{id}:=E \\
& \text { id }:=\underline{E} ; \text { id }:=E \\
& \text { id }:=\text { num ;id }:=\underline{E} \\
& \text { id }:=\text { num ;id }:=E+\underline{E} \\
& \text { id }:=\text { num ;id }:=\underline{E}+(S, E) \\
& \text { id }:=\text { num } ; \mathrm{id}:=\mathrm{id}+(\underline{S}, E) \\
& \text { id }:=\text { num } ; \mathrm{id}:=\mathrm{id}+(\mathrm{id}:=\underline{E}, E) \\
& \text { id }:=\text { num } ; \mathrm{id}:=\mathrm{id}+(\mathrm{id}:=E+E, \underline{E}) \\
& \text { id }:=\text { num ;id }:=\mathrm{id}+(\mathrm{id}:=\underline{E}+E, \text { id }) \\
& \text { id }:=\text { num } ; \mathrm{id}:=\mathrm{id}+(\mathrm{id}:=\text { num }+\underline{E}, \text { id }) \\
& \text { id }:=\text { num } ; \mathrm{id}:=\mathrm{id}+(\mathrm{id}:=\text { num }+ \text { num }, \text { id })
\end{aligned}
$$

## Ambiguity

Consider the following straight-line program.


- Does the right-hand side denote $(b+b)+c$ or $b+(b+c)$ ?
- Does it matter?
a language with expressions made up of terms and factors.

1. $E \rightarrow E+T$
2. $E \rightarrow E-T$
3. $E \rightarrow T$
4. $T \rightarrow T * F$
5. $T \rightarrow T / F$
$T \rightarrow F$
6. $F \rightarrow \mathrm{id}$
7. $F \rightarrow$ num
8. $F \rightarrow(E)$

## Parsing by recursive descent

Consider the following grammar.

1. $S \rightarrow$ if $E$ then $S$ else $S$
2. $S \rightarrow$ begin $S L$
3. $S \rightarrow$ print $E$
4. $L \rightarrow$ end
5. $L \rightarrow$; $S L$
6. $E \rightarrow$ num $=$ num

This grammar can be parsed using a simple algorithm which is known as recursive descent. A recursive descent parser has one function for each non-terminal and one clause for each production.

## First and follow sets

Grammars consist of terminals and non-terminals. With respect to a particular grammar, given a string $\gamma$ of terminals and non-terminals,

- nullable $(X)$ is true if $X$ can derive the empty string
- $\operatorname{FIRST}(\gamma)$ is the set of terminals that can begin strings derived from $\gamma$
- $\operatorname{FOLLOW}(X)$ is the set of terminals that can immediately follow $X$. That is, $t \in \operatorname{FOLLOW}(X)$ if there is any derivation containing $X t$. This can occur if the derivation contains $X Y Z t$ where $Y$ and $Z$ both derive $\epsilon$.


## Computing FIRST, FOLLOW and nullable

Initialise FIRST and FOLLOW to all empty sets
Initialise nullable to all false.
for each terminal symbol $Z$
$\operatorname{FIRST}[Z] \leftarrow\{Z\}$
repeat
for each production $X \leftarrow Y_{1} Y_{2} \cdots Y_{k}$
for each $i$ from 1 to $k$, each $j$ from $i+1$ to $k$
if all the $Y_{i}$ are nullable
then nullable $[X] \leftarrow$ true
if $Y_{1} \cdots Y_{i-1}$ are all nullable
then $\operatorname{FIRST}[X] \leftarrow \operatorname{FIRST}[X] \cup \operatorname{FIRST}\left[Y_{i}\right]$
if $Y_{i+1} \cdots Y_{k}$ are all nullable
then FOLLOW $\left[Y_{i}\right] \leftarrow$ FOLLOW $\left[Y_{i}\right] \cup$ FOLLOW $[X]$
if $Y_{i+1} \cdots Y_{j-1}$ are all nullable
then FOLLOW $\left[Y_{i}\right] \leftarrow$ FOLLOW $\left[Y_{i}\right] \cup$ FIRST $\left[Y_{j}\right]$
until FIRST, FOLLOW and nullable did not change in this iteration

```
enum token tok;
void advance() {tok=getToken();}
void eat(enum token t) {if (tok==t) advance(); else error();}
void S(void) {switch(tok) {
    case IF: eat(IF); E(); eat(THEN); S();
                                    eat(ELSE); S(); break;
    case BEGIN: eat(BEGIN); S(); L(); break;
    case PRINT: eat(PRINT); E(); break;
    default: error();
    }}
void L(void) {switch(tok) {
    case END: eat(END); break;
        case SEMI: eat(SEMI); S(); L(); break;
    default: error();
    }}
void E(void) { eat(NUM); eat(EQ); eat(NUM); }
```


## When recursive descent fails

```
S->E$ E->E+T T->T*F\quadF->\textrm{id}
E->E-T T}->T/F\quadF->\mathrm{ num
    E->T T}->F\quadF->(E
void S(void) { E(); eat(EOF); }
void E(void) {switch(tok) {
        case ?: E(); eat(PLUS); T(); break;
        case ?: E(); eat(MINUS); T(); break;
        case ?: T(); break;
        default: error();
    }}
void T(void) {switch(tok) {
        case ?: T(); eat(TIMES); F(); break;
        case ?: T(); eat(DIV); F(); break;
        case ?: F(); break;
        default: error();
    }}
```


## Constructing a predictive parser

- The information which we need can be coded as a two-dimensional table of productions, indexed by non-terminals and terminals. This is a predictive parsing table.
- To construct the table, enter the production $X \rightarrow \gamma$ in column $T$ of row $X$ for each $T \in \operatorname{FIRST}(\gamma)$. Also, if $\gamma$ is nullable, enter the production in column $T$ of row $X$ for each $T \in \operatorname{FOLLOW}(X)$.
- An ambiguous grammar will always lead to some locations in the table having more than one production.
- A grammar whose predictive parsing table has at most one production in each location is called LL(1). This stands for Left-to-right parse, leftmost derivation, 1-symbol lookahead.


## Detecting ambiguity with a parsing table

$$
\begin{array}{lll}
Z \rightarrow d & Y \rightarrow & X \rightarrow Y \\
Z \rightarrow X Y Z & Y \rightarrow c & X \rightarrow a
\end{array}
$$

|  | nullable | FIRST | FOLLOW |
| :---: | :---: | :---: | :---: |
| $X$ | true | $\{a, c\}$ | $\{a, c, d\}$ |
| $Y$ | true | $\{c\}$ | $\{a, c, d\}$ |
| $Z$ | false | $\{a, c, d\}$ |  |


|  | $a$ | $c$ | $d$ |
| :---: | :---: | :---: | :---: |
| $X$ | $X \rightarrow a$ | $X \rightarrow Y$ | $X \rightarrow Y$ |
| $Y$ | $X \rightarrow Y$ | $Y \rightarrow$ | $Y \rightarrow$ |
| $Z$ | $Y \rightarrow$ | $Y \rightarrow c$ | $Z \rightarrow d$ |
| $Z$ | $Z \rightarrow X Y Z$ | $Z \rightarrow X Y Z$ | $Z \rightarrow X Y Z$ |


$E \rightarrow T$
$T \rightarrow F$
$F \rightarrow(E)$
We can transform this grammar as shown below.

$$
\begin{array}{llll}
S \rightarrow E \$ & E \rightarrow T E^{\prime} & T \rightarrow F T^{\prime} & F \rightarrow \mathrm{id} \\
& & & F \rightarrow \text { num } \\
& E^{\prime} \rightarrow+T E^{\prime} & T^{\prime} \rightarrow * F T^{\prime} & F \rightarrow(E) \\
& E^{\prime} \rightarrow T E^{\prime} & T^{\prime} \rightarrow / F T^{\prime} & \\
& E^{\prime} \rightarrow & T^{\prime} \rightarrow &
\end{array}
$$

## Left factoring a grammar

Left recursion interferes with predictive parsing. A similar problem occurs when two productions for the same non-terminal start with the same symbols. For example:

$$
\begin{aligned}
& S \rightarrow \text { if } E \text { then } S \text { else } S \\
& S \rightarrow \text { if } E \text { then } S
\end{aligned}
$$

In such a case we can left-factor the grammar to account for the optional else statement.

$$
\begin{aligned}
S & \rightarrow \text { if } E \text { then } S X \\
X & \rightarrow \\
X & \rightarrow \text { else } S
\end{aligned}
$$

\}\}
int Tprime_follow [] = \{PLUS, MINUS, RPAREN, EOF, -1\};

```
void Tprime(void) { switch (tok) {
    case PLUS: break;
    case MINUS: break;
    case TIMES: eat(TIMES); F(); Tprime(); break;
    case DIV: eat(DIV); F(); Tprime(); break;
    case RPAREN: break;
    case EOF: break;
    default: printf("expected +, -, *, /, ) or EOF");
}}
```


## LR Parsing

- The weakness of $\mathrm{LL}(k)$ parsing techniques is that they must predict which production to use, having seen only the first $k$ tokens of the right-hand side.
- LR $(k)$ parsing postpones the decision until it has seen the entire right-hand side of the production (and $k$ more tokens beyond).
- The name $\mathrm{LR}(k)$ means Left-to-right parse, rightmost derivation, $k$-token lookahead.
- The parser has a stack and an input. The first $k$ tokens of the input are the lookahead. The parser performs two kinds of actions:

Shift: move the first input token to the top of the stack;
Reduce: Choose a grammar rule $X \rightarrow A B C$; pop $C, B, A$ off the stack, push $X$ onto the stack.

- Shifting the end-of-file marker is called accepting.


## Transition table generation

- An $\operatorname{LR}(k)$ parser uses the contents of its stack and the next $k$ tokens of the input to decide which action to take. In practice the value of $k$ is 1

> - because transition tables for $k>1$ would be huge; and
> - most programming languages have $\operatorname{LR}(1)$ grammars.

- To construct a transition table for an $\operatorname{LR}(0)$ parser we need to explore the possible states of the parser. For this, we use the notion of an item (such as an $\operatorname{LR}(0)$ item or an $\operatorname{LR}(1)$ item). An item is a grammar rule, combined with a dot which indicates a position in its right hand side. For example:

$$
S^{\prime} \rightarrow . S \$
$$

A state is just a set of items.

## An example of $\operatorname{LR}(0)$ items and states

$$
\begin{array}{lll}
S^{\prime} \rightarrow S \$ & S \rightarrow(L) & L \rightarrow S \\
& S \rightarrow r & L \rightarrow I
\end{array}
$$

We build up states starting from the start symbol $S^{\prime}$. There are nine states in total and the first six are these.

for any production $X \rightarrow \gamma$

$$
I \leftarrow I \cup\{X \rightarrow . \gamma\}
$$

until $I$ does not change
return $I$
/*Moving past a non-terminal $X$ */
$\operatorname{Goto}(I, X)=$
set $J$ to the empty set
for any item $A \rightarrow \alpha \cdot X \beta$ in $I$
add $A \rightarrow \alpha X . \beta$ to $J$
return Closure $(J)$

## LR(0) parser construction

/*Compute the states traversed ( $T$ ) and edge set (E) */
Initialize $T$ to $\left\{\right.$ Closure $\left.\left(\left\{S^{\prime} \rightarrow . S \$\right\}\right)\right\}$
Initialize $E$ to the empty set.

## repeat

for each state $I$ in $T$
for each item $A \rightarrow \alpha . X \beta$ in $I$
let $J$ be $\operatorname{Goto}(I, X)$
$T \leftarrow T \cup\{J\}$

$$
E \leftarrow E \cup\{I \xrightarrow{X} J\}
$$

until $E$ and $T$ did not change in this iteration

$$
\begin{aligned}
& /{ }^{*} \text { Compute the set of } L R(0) \text { reduce actions }(R) * / \\
& R \leftarrow\} \\
& \text { for each state } I \text { in } T \\
& \quad \text { for each item } A \rightarrow \alpha \text {. in } I \\
& \quad R \leftarrow R \cup\{(I, A \rightarrow \alpha)\}
\end{aligned}
$$

## Using parser generators

- The task of constructing $\operatorname{LR}(1)$ and $\operatorname{LALR}(1)$ parsing tables is simple enough to be automated.
- Many such parser generator tools exist (Yacc, Bison, ML-Yacc, CUP, JavaCC, ...)
- Grammar specifications are divided into sections (the following is for Yacc)
parser declarations
$\% \%$
grammar rules
$\% \%$
programs
- The grammar rules are productions of the following form.

$$
\exp : \exp \text { PLUS } \exp \quad\{\text { semantic action }\}
$$

## An example grammar

1. $P \rightarrow L$
2. $S \rightarrow$ id $:=\mathrm{id}$
(programs)
3. $S \rightarrow$ while id do $S$
4. $S \rightarrow$ begin $L$ end
5. $S \rightarrow$ if id then $S$
6. $S \rightarrow$ if id then $S$ else $S$
7. $L \rightarrow S$
8. $L \rightarrow L ; S$
(assignment statements)
(while loops)
(compound statements)
(if-then statements)
(if-then-else statements)
(singleton statement list)
(non-singleton statement list)

- For each state $I$ containing $S^{\prime} \rightarrow S . \$$ we put Accept at $(I, \$)$.
- For each state $I$ containing $A \rightarrow \gamma$. (production $n$ with the dot at the end), we put a Reduce $(n)$ action at $(I, Y)$ for every token $Y$.


## Parsers which are better than LR(0)

SLR (Simple LR) parsers
$/ *$ Compute the set of SLR reduce actions ( $R$ ) */
$R \leftarrow\}$
for each state $I$ in $T$
for each item $A \rightarrow \alpha$. in $I$
for each token $X$ in $\operatorname{Follow}(A)$

$$
R \leftarrow R \cup\{(I, X, A \rightarrow \alpha)\}
$$

## LR(1) parsers

/*Items now include a one-symbol lookahead */
$R \leftarrow\}$
for each state $I$ in $T$
for each item $(A \rightarrow \alpha ., z)$ in $I$
$R \leftarrow R \cup\{(I, z, A \rightarrow \alpha)\}$

## Implementation in Yacc/Bison

\% \{
int yylex(void)
void yyerror (char *s) \{ EM_error(EM_tokPos, "\%s", s); \} \%\}
\%token ID WHILE BEGIN END DO IF THEN ELSE SEMI ASSIGN \%start prog
\% \%
prog: stmlist
stm : ID ASSIGN ID
| WHILE ID DO stm
| BEGIN stmlist END
I IF ID THEN stm
| IF ID THEN stm ELSE stm
stmlist : stm
| stmlist SEMI stm

## Implementation in CUP

terminal ID, WHILE, BEGIN, END, DO, IF, THEN, ELSE, SEMI, ASSIGN;
non terminal prog, stm, stmlist;
start with prog;
prog ::= stmlist;
stm ::= ID ASSIGN ID
WHILE ID DO stm
| BEGIN stmlist END
| IF ID THEN stm
I IF ID THEN stm ELSE stm;
stmlist ::= stm
| stmlist SEMI stm;

- A shift-reduce conflict is a choice between shifting and reducing.
- A reduce-reduce conflict is a choice of reducing by two different rules.
- Two conventions apply:
- Resolve shift-reduce conflicts by shifting.
- Resolve reduce-reduce conflicts by using the rule which
appears earliest in the grammar.
- Causes of conflicts can be found by examining the verbose description file.


## LR parsing of ambiguous grammars

Many programming languages have grammar rules such as

$$
\begin{aligned}
& S \rightarrow \text { if } E \text { then } S \text { else } S \\
& S \rightarrow \text { if } E \text { then } S
\end{aligned}
$$

which allow programs such as
if a then if b then s 1 else s 2
Such a program could be understood in two ways

1. if a then \{ if b then s1 else s2 \}
2. if a then \{ if b then s1 \} else s2

The grammar leads to a shift/reduce conflict, resolved by shifting.

```
\(E \rightarrow\) int \(\quad E \rightarrow E+E \quad E \rightarrow E * E \quad E \rightarrow E-E \quad E \rightarrow-E\)
```

| $\%\{$ declarations of yylex and yyerror \%\} \%token INT PLUS MINUS TIMES UMINUS |  |
| :---: | :---: |
|  |  |
| \%token INT PLUS MINUS TIMES UMINUS $\%$ start exp |  |
| \%left PLUS MINUS | /* Left associativity. */ |
| \%left TIMES | /* Left associativity, higher precedence. */ |
| \%left UMINUS | /* Left associativity, highest precedence. */ |
| \%\% |  |
| exp : int |  |
| 1 exp PLUS exp | /* By default, a rule has the precedence */ |
| 1 exp MINUS exp | /* of its last (i.e. rightmost) token. */ |
| 1 exp TIMES exp |  |
| 1 MINUS exp \%prec UMINUS | /* Explicitly setting precedence. |

## Non-associative operators

- Some programming languages consider it unnecessarily confusing to allow some operators to be either left associative or right associative. Thus $a-b-c$ might be considered to be grammatically ill-formed.
- The Pascal programming language disallowed the <= operator on Booleans from being associative. The ordering which Pascal placed on Booleans was false < true. Because the operator is non-associative, the following Boolean expression is illegal.

```
false <= false <= false
```

- Non-associativity can be specified with the \%nonassoc directive.


## A simple interpreter in Yacc/Bison

```
%{ declarations of yylex and yyerror %}
%union {int num; string id;}
%token <num> INT
%token <id> ID
%type <num> exp
%start exp
%left PLUS MINUS
%left TIMES
%left UMINUS
%%
exp : INT {$$ = $1;}
    | exp PLUS exp {$$ = $1 + $3;}
    | exp MINUS exp {$$ = $1 - $3;}
    | exp TIMES exp {$$=$1*$3;}
    | MINUS exp %prec UMINUS {$$=-$2;}
```


## Parse trees

- Compilers do not evaluate programs as interpreters do. A compiler subjects the program text to semantic analysis and then translates into a lower-level language. A parse tree is used to store the program text for analysis and translation purposes.
- A parse tree for the concrete syntax of the language would have exactly one leaf for each token of the input and one internal node for each grammar rule reduced during the parse.
- A parse tree for the abstract syntax retains the important content from the concrete parse tree but dispenses with the punctuation tokens which were needed to ensure the correct parse.
- The resulting language can be described by a smaller, simpler grammar.
$\begin{array}{lr}\text { 2. } S \rightarrow \text { id }:=E & \text { (assignment statements) } \\ \text { 3. } S \rightarrow \text { print } L & \text { (print statements) } \\ \text { 4. } E \rightarrow \mathrm{id} & \text { (identifier usage) } \\ \text { 5. } E \rightarrow \text { num } & \text { (numerical values) } \\ \text { 6. } E \rightarrow E B E & \text { (binary operator use) } \\ \text { 7. } E \rightarrow S, E & \text { (comma expressions) } \\ \text { 8. } L \rightarrow E & \text { (singleton lists) } \\ \text { 9. } L \rightarrow L E & \text { (non-singleton lists) } \\ \text { 10. } B \rightarrow+ & \text { (addition) } \\ \text { 11. } B \rightarrow- & \text { (subtraction) } \\ \text { 12. } B \rightarrow * & \text { (multiplication) } \\ \text { 13. } B \rightarrow / & \text { (division) }\end{array}$


## Error reporting and positional information

- In a one-pass parser, lexical analysis, parsing and semantic analysis are done simultaneously. If there is a type-checking error then the current position of lexer can be used in diagnostic error messages.
- A modern compiler makes many passes through the program so the parse tree must be sprinkled with pos fields. These indicate the position, within the original source file, of the characters from which the abstract syntax structures were derived.
- Ideally the parser should maintain a position stack along with the semantic value stack. Bison provides this; Yacc does not.

```
struct {
    int first_line, last_line;
    int first_column, last_column;
};
```


## Symbol tables

- The semantic analysis phase of a compiler is characterised by the maintenance of symbol tables (also called environments) which map identifiers used in the source program to their types and locations.
- As declarations of types, variables and functions are processed by the compiler the identifiers used are bound to "meanings" in the symbol table. An environment is a set of bindings.
- When uses (non-defining occurrences) of the identifiers are found, the symbol table is consulted to check the suitability of the use.
- Each local variable in a program has a scope in which it is visible. As the semantic analysis reaches the end of each scope, the bindings local to that scope are discarded.


## Environments in Tiger

Consider the introduction and elimination of environments in the small Tiger program given below. This program is being compiled in the environment $\sigma_{0}$.

```
let var a := 10
    \sigma}=\mp@subsup{\sigma}{1}{}+{\textrm{b}\mapsto\mathrm{ int }
    in print_int(b);
    let var j := a+b
        \sigma
        in print(a); print_int(j)
    end;
    print_int(a)
end
```

(a :=5; a+1)
translates into abstract syntax as shown below.

```
A_SeqExp(2,
    A_Exp_List (A_AssignExp(4,
        A_SimpleVar(2,S_Symbol("a")),
        A_IntExp \((7,5))\),
    A_ExpList(A_OpExp(11,
        A_plusOp,
        A_VarExp(A_SimpleVar(10, S_Symbol("a"))),
        A_IntExp \((12,1))\),
    NULL)))
```


## Derived forms/defined notation

- The abstract syntax of a language can be further compressed by defining some constructs of the language in terms of others.

$$
\begin{aligned}
& -e_{1} \& e_{2} \text { is translated to if } e_{1} \text { then } e_{2} \text { else } 0 \\
& -e_{1} \mid e_{2} \text { is translated to if } e_{1} \text { then } 1 \text { else } e_{2} \\
& --i \text { is translated as }(0-i)
\end{aligned}
$$

- Derived forms lead to simplifications in the semantic analysis process but they make it harder to generate good diagnostic error messages for semantic errors.


## Multiple environments in ML and Java

```
structure M = struct
    structure E = struct
        val a = 5
    end
    structure N = struct
        val b = 10
        val a = E.a + b
    end
    structure D = struct
        val d = E.a + N.a
    end
end
```


## Multiple symbol tables

With many environments active at once, we must associate names with environments. This leads to the following collection of symbol tables.


```
package M;
class E {
    static int a = 5;
}
class N {
    static int b = 10;
    static int a = E.a + b;
}
class D {
    static int d = E.a + N.a;
}
```

$$
\left\{\begin{array}{l}
\sigma_{1}=\{\mathrm{a} \mapsto \text { int }\} \\
\sigma_{2}=\left\{\mathrm{E} \mapsto \sigma_{1}\right\} \\
\sigma_{3}=\{\mathrm{b} \mapsto \text { int }, \mathrm{a} \mapsto \text { int }\} \\
\sigma_{4}=\left\{\mathrm{N} \mapsto \sigma_{3}\right\} \\
\sigma_{5}=\{\mathrm{d} \mapsto \text { int }\} \\
\sigma_{6}=\left\{\mathrm{D} \mapsto \sigma_{5}\right\} \\
\sigma_{7}=\sigma_{2}+\sigma_{4}+\sigma_{6} \\
\sigma_{8}=\sigma_{0}+\left\{\mathrm{M} \mapsto \sigma_{7}\right\}
\end{array}\right.
$$

let type $\mathrm{a}=\{\mathrm{x}:$ int, y : int $\}$
type b = \{ x: int, y : int \}
var i : a := ...
var j : b := ...
in $\mathrm{i}:=\mathrm{j}$
end
If we were to change the language to allow usage such as this we would have to examine record types field by field, recursively, when testing for equality. This would slow down the type-checking phase of the compilation considerably.

## Record types in Tiger

The following program is legal.

```
let type a = { x: int, y: int }
    type c = a
    var i : a := ...
    var j : c := ...
    in i := j
end
```

It is the type expression which causes a new type to be made; not the type declaration.

## Special cases

- In Tiger the expression nil belongs to any record type. We handle this by inventing a special 'nil' type.
- There are expressions in Tiger which return no value (such as the assignment expression) so we invent a 'void' type.
- When processing mutually recursive types we need a place holder for types whose name we know but whose definition we have not yet seen. We enter a null pointer for the type name to come and then later replace it.
abstract syntax of the language. It contains these functions.

```
struct expty transVar(S_table venv, S_table tenv, A_var v);
struct expty transExp(S_table venv, S_table tenv, A_exp a);
void transDec(S_table venv, S_table tenv, A_dec d);
struct Ty_ty transTy ( S_table tenv, A_ty a);
```

The expty type contains a translated expression and its Tiger-language type.

```
struct expty {Tr_exp exp; Ty_ty ty;};
struct expty expTy(Tr_exp exp, Ty_ty ty) {
    struct expty e; e.exp=exp; e.ty=ty; return e;
```

\}

## Type-checking arithmetic expressions (in ML)

For a type identifier we need to remember the type that it denotes. The type environment is initialised to include definitions for int and string.

- For each value identifier we need to know whether it is a variable or a function. If it is a variable we need to know its type. If it is a function we need to know its parameter and result types. The variable environment is initialised to include definitions of predefined functions.
- As types, variables and functions are declared, the type-checker augments the environments. The environments are consulted for each identifier found when processing expressions.


## Type-checking expressions

Type-checking is the first part of the translation process which we consider. Our semantic analysis module performs type-checking of the

```
fun transExp(venv, tenv,
```

fun transExp(venv, tenv,
Absyn.OpExp{left,oper=Absyn.PlusOp,right,pos}) =
Absyn.OpExp{left,oper=Absyn.PlusOp,right,pos}) =
let val {exp=_, ty=tyleft} = transExp(venv, tenv, left)
let val {exp=_, ty=tyleft} = transExp(venv, tenv, left)
val {exp=_, ty=tyright} = transExp(venv, tenv, right)
val {exp=_, ty=tyright} = transExp(venv, tenv, right)
in case tyleft of Types.INT => ()
in case tyleft of Types.INT => ()
| _ => error pos "integer required";
| _ => error pos "integer required";
case tyright of Types.INT => ()
case tyright of Types.INT => ()
of Types.INT => ()
of Types.INT => ()
{exp=(), ty=Types.INT}
{exp=(), ty=Types.INT}
end

```
    end
```

```
    \vdots
    case A_opExp: {
        A_oper oper = a->u.op.oper;
        struct expty left = transExp(venv, tenv, a->u.op.left);
        struct expty right= transExp(venv, tenv, a->u.op.right);
            if (oper==A_plusOp) {
            if (left.ty.kind != Ty_int)
                EM_error(a->u.op.left->pos, "integer required");
            if (right.ty.kind != Ty_int)
                EM_error(a->u.op.right->pos, "integer required");
            return expTy(NULL, Ty_int());
        }
    }
    }
    }
}
```


## Type-checking variables (in Java)

```
ExpTy transVar(Absyn.SimpleVar v) {
    Entry x = (Entry)env.venv.get(v.name);
    if (x instanceof VarEntry) {
        /* found the variable in the symbol table */
        VarEntry ent = (VarEntry)x;
        return new ExpTy (null, ent.ty);
    }
    else {
        /* failed to find the variable */
        error (v.pos, "undefined variable");
        return new ExpTy(null, INT);
    }
```

\}

## Type-checking variable declarations

```
void transDec(S_table venv, S_table tenv, A_dec d) {
    switch(d->kind) {
        case A_varDec: {
            /* var id := exp */
            struct expty e = transExp(venv,tenv,d->u.var.init);
            S_enter(venv, d->u.var.var, E_VarEntry(e.ty));
        }
    }
}
```

If a type constraint is present it would also be necessary to check that the constraint and the expression are compatible. Initialisations to nil must be accompanied by a type constraint which is of a record type.

## Type-checking function declarations

```
void transDec(S_table venv, S_table tenv, A_dec d) {
    switch(d->kind) {
    case A_functionDec: {
        A_fundec f = d->u.function->head;
        Ty_ty resultTy = tylook(tenv,f->result,f->pos);
        Ty_tyList formalTys = makeFormalTyList(tenv,f->params);
        /* for non-recursive functions only */
        S_enter(venv,f->name,E_FunEntry(formalTys,resultTy));
        S_beginScope(venv);
        { A_fieldList l; Ty_tyList t;
            for(l=f->params, t=formalTys; l; l=l->tail, t=t->tail)
                    S_enter(venv,l->head->name,E_VarEntry(t->head));
        }
        /* should check compatibility with result type */
        transExp(venv, tenv, d->u.function->body);
        S_endScope(venv);
        break;
    }
```

case A_simpleVar:
E_enventry x = S_look(venv,v->u.simple);
if ( $x$ \&\& $x$->kind==E_varEntry)
/* found the variable in the symbol table */
/* found the variable in the symbol table */
return expTy(NULL, actual_ty(x->u.var.ty));
else
/* failed to find the variable */
\{ EM_error(v->pos, "undefined variable \%s",
S_name(v->u.simple));
return expTy(NULL, Ty_int()); $\}$
\}
case A_fieldVar
$\vdots$
\}
$\}$

## Type-checking let-expressions

```
struct expty transExp(S_table venv, S_table tenv, A_exp a) {
switch(a->kind) {
    \vdots
    case A_letExp: {
        /* let d in exp end */
        struct expty exp;
        A_declist d;
        A_declist d;
        S_beginScope(venv);
        /* process the declarations in the list */
        for (d = a->u.let.decs; d; d = d->tail)
            transDec(venv, tenv, d->head);
        /* translate the expression in the body */
        exp = transExp(venv, tenv, a->u.let.body);
        S_endScope(tenv);
        S_endScope(venv);
        return exp;
    }
}
```


## Recursive declarations

- When type-checking a collection of mutually recursive declarations the compiler must make two passes through the declarations.
- In the first pass the aim is to collect all of the headers of the declarations.
- The header of a type is its identifier.
- The header of a function is its name, formal parameter list and return type.
- In the second pass the bodies of the declarations are processed in the environment extended with the header information.


## Local variables

The following function has a parameter x and a local variable y .

```
function f(x : int) : int =
    let var y := x+x
    in if y < 10
            then f(y)
            else y-1
    end
```

When the function $f$ is called a new instantiation of $x$ is created and initialised by f's caller. Because there are recursive calls to $f$ many of these x's exist simultaneously. Similarly, many y's exist simultaneously. If local variables are created on function entry and destroyed on function exit, then we can use a stack to hold them.
valued variables it may be necessary to keep local variables after a
function call has returned control to its caller. Consider the following Standard ML function.

```
fun f x =
    let fun g y = x+y
    in g
    end
```

When $f$ returns it is too soon to destroy $x$. The value which $x$ was given in the call to $f$ will be needed when the function returned by $f$ is called.

In order to be able to use the stack discipline to store local variables most programming languages provide either nested functions (Pascal and Tiger) or functions as returnable results (C) but not both.

## Stack frames

- We will consider functions whose local variables can be stored on a stack. We treat the stack as a big array with a stack pointer. All locations past the stack pointer are considered to contain rubbish and all of the locations before it are considered to be allocated.
- The stack grows on entry to a function by an increment large enough to hold all of the local variables. On exit it shrinks by the same amount.
- The area on the stack devoted to local variables, parameters, return address and other temporary values is called the function's activation record or stack frame.
- Sometimes the manufacturer of a computer prescribes a "standard" frame layout. Using this allows calls to functions written in other programming languages.


## Function calling conventions

- A machine usually has only one set of registers, which many different functions need to use. Suppose $f$ is using register $r$ and calls $g$, which also uses $r$. Then $r$ must be saved (into a stack frame) before $g$ uses it and restored afterwards.
- If it is $f$ 's responsibility to do this then we say that $r$ is a callersave register. If it is $g$ 's responsibility then we say that $r$ is a callee-save register.
- On most architectures this notion is a convention described in the machine's reference manual. On the MIPS architecture registers 16 to 23 are callee-save and all others are not.
- Sometimes the saves and restores are unnecessary. If $f$ knows that the value of some local variable in not needed after the call it can put it in a caller-save register and then not save it. The selection of the appropriate kind of register for each variable is done by the register allocator routine.


Studies of actual programs have shown that very few functions have more than four arguments, and almost none have more than six.

Therefore, parameter-passing conventions for modern machines specify that the first $k$ arguments ( $k=4$ or $k=6$, typically) of a function are passed in registers and the rest in memory.

However, suppose that $f\left(a_{1} 1, \ldots, a \_n\right)$ calls $h(z)$. It must pass $z$ in $r_{-} 1$ (say) and save $a_{-} 1$. How has the use of registers saved any time?

- On entry to $f$, the stack pointer points to the first argument which $g$ passes to $f$. On entry, $f$ allocates a frame by subtracting the frame size from the stack pointer SP. The old SP becomes the current frame pointer FP.
- If the frame size is fixed, then for each function $f$ the FP will always differ from SP by a known constant, and it is not necessary to use a register for FP at all- FP is a 'virtual' register whose value is always $\mathrm{SP}+$ framesize.


## Registers

- A modern machine has a large set of registers (typically 32 of them). For efficiency reasons, it is useful to keep local variables, intermediate results of expressions, and other values in registers instead of in the stack frame.
- The benefit of doing this is that registers can be directly accessed by arithmetic instructions whereas accessing memory typically requires load and store instructions.
- Even on those machines whose arithmetic instructions can access memory, it is still faster to access registers.


## Boring Fact about $\mathfrak{P r o g r a m s ~ \# 2 ~}$

Most functions which are called are leaf functions.
That is, they do not call any others

1. Leaf functions need not store their incoming arguments in memory. In fact, they often do not need a stack frame at all.
2. Some optimising compilers use interprocedural register allocation, analysing all the functions in a program at once.
3. The threatened variable may now actually be dead anyway, and need not be saved.
4. Some architectures have register windows so that each function invocation can allocate a fresh set without memory traffic.

## Address accesses and pointer arithmetic

- C allows programmers to take the address of a formal parameter and guarantees that all the formal parameters of a function are at consecutive addresses. The later fact is used by printf.
- Allowing programmers to take addresses can lead to dangling references.


## int $*$ f (int $x$ ) \{ return \&x; \}

- To resolve the contradiction that parameters are passed in registers but have addresses in memory too, the first $k$ of them are passed in registers but any parameter whose address is taken must be written to a memory location on entry to the function.
- Because the memory locations into which register arguments are written must be consecutive if the addresses of parameters 1 and 5 are taken in the function then parameters 2,3 and 4 must be written to memory on entry to the function also.
to use call-by-reference. With call-by-reference, the programmer does not explicitly manipulate the address of a variable, $x$. Instead, if $x$ is passed to $f(y)$ where $y$ is a "by reference" parameter, the compiler generates code to pass the address of $x$ instead of its contents.
- At any use of $y$ in the function body the compiler generates an extra pointer dereference.
- With call-by-reference, there are no dangling references because $y$ must disappear when $f$ returns.
- The Ada programming language has three types of parameter passing, input, output and transput. Requiring the programmer to be so discerning allows the compiler to generate good code for function calls with no copying of values.


## Return addresses

- When function $g$ calls function $f$, eventually $f$ must return. It needs to know where to go back to.
- If the call instruction within $g$ is at address $a$, then usually the right place is address $a+1$. This is called the return address.
- The call instruction puts the return address in a designated register. A non-leaf function will then have to write it to the stack (unless interprocedural analysis says otherwise) whereas a leaf function will not.


## A nested functions example

```
type tree = {key : string, left: tree, right: tree}
function prettyprint (tree: tree) : string =
let var output := ""
    function write (s: string) =
        output := concat (output, s)
    function show (n: int, t: tree) =
        let function indent (s: string) =
            (for i := 1 to n do write (" ");
                output := concat (output, s))
            in if t=nil then indent (".")
            else (indent (t.key);
                show (n+1, t.left);
                show (n+1, t.right))
            end
in show (0, tree); output
end
```


## Abstracting from machine specific details

- Different target machine architectures will have different notions of stack frames. In order to produce a portable compiler we should abstract away from the details of the standard layout on a particular architecture.
- We define an interface for the routines which will process stack frames (frame.h). If machine-independent parts of the compiler only manipulate frames through this abstract interface then we will be able to replace one implementation of frames with another.
traffic, values are written to memory (in the stack frame) only when
necessary for one of these reasons.

1. The variable will be passed by reference, or the \& operator applied.
2. The variable is accessed by a nested procedure.
3. The variable is too big to fit in a single register.
4. The variable is an array, necessitating address arithmetic.
5. The register holding the variable is needed for a particular purpose.

6 . There are so many local variables and parameters that they must be "spilled" into the frame.

We say that a variable escapes if it is passed by reference, its address is taken (using \&), or it is accessed from a nested function.

## Static links

- In languages which allow nested functions, an inner function may use variables declared in an outer one. This language feature is called block structure.
- In such languages functions may have to have references to not only their own frame but also those of outer functions. There are several methods to accomplish this.
- Whenever $f$ is called it is passed a pointer to the frame of the function statically enclosing $f$. This pointer is the static link.
- A global array can be maintained containing-in position $i$ a pointer to the frame of the most recently entered procedure whose static nesting depth is $i$. Such an array is called a display.
- When $g$ calls $f$, each variable of $g$ that is actually accessed by $f$ is actually passed as an extra argument. This is called lambda lifting.


## An interface to frames

```
/* frame.h */
typedef struct F_frame_ *\underline{F_frame;}
typedef struct F_access_ *F_access; /* an abstract
    data type for parameters or local variables */
typedef struct F_accessList_ *F_accessList;
struct F_accessList_ { F_access head; F_accessList tail;};
F_frame F_newFrame(Temp_label name, U_boolList formals);
Temp_label F_name(F_frame f);
F_accessList F_formals(F_frame f);
F_access F_allocLocal(F_frame f, bool escape);
    \vdots
```


## Frame creation and use

- The type F_frame holds information about formal parameters and local variables. To make a new frame for a function $f$ with $k$ formal parameters, call F_newFrame $(f, l)$, where $l$ is a list of $k$ booleans recording whether or not each parameter escapes. E.g.

> F_newFrame ( $g$, U_BoolList (TRUE,
> U_BoolList (FALSE,
> U_BoolList (FALSE, NULL))))

- The F_formals interface function returns a list of $k$ accesses denoting the locations where the formal parameters will be kept at run time, as seen from inside the callee.
- The difference in the caller's method of access and the callee's is called the view shift.

```
struct F_access_
    { enum { inFrame, inReg } kind;
        union {
                int offset; /* InFrame */
                Temp_temp reg; /* InReg */
        } u;
    };
```

    static F_access InFrame(int offset);
    static F_access InReg(Temp_temp reg);
    InFrame $(X)$ indicates offset $X$ from the frame pointer; $\operatorname{InReg}\left(t_{84}\right)$ indicates that it will be held in register $t_{84}$.

## The view shift

- Because the "view shift" (from the caller's perspective to the callee's perspective) depends on the calling conventions of the target machine it must be handled by the Frame module, starting with newFrame.
- For each formal parameter, newFrame must calculate two things:
- How the parameter will be seen from inside the function (in a register, or in a frame location);
- What instructions must be produced to implement the "view shift"
- For example, a frame-resident parameter will be seen as "memory at offset $X$ from the frame pointer" and the view shift will be implemented by copying the stack pointer to the frame pointer on entry to the procedure.

$$
\operatorname{InFrame}(0) \quad \operatorname{InReg}\left(t_{157}\right) \quad \operatorname{InReg}\left(t_{158}\right)
$$

- The view shift could be implemented in this way:

$$
\begin{aligned}
& \mathrm{sp} \leftarrow \mathrm{sp}-K \\
& M[\mathrm{sp}+K+0] \leftarrow \mathrm{r} 2 \\
& t_{157} \leftarrow \mathrm{r} 4 \\
& t_{158} \leftarrow \mathrm{r} 5
\end{aligned}
$$

- If the register allocator ultimately chooses $r 4$ for the second parameter and r 5 for the third then the unnecessary move instructions at the end of the view shift could be deleted at that time.


## Local variables

- Some local variables are kept in the frame; others are kept in registers. To allocate a new local variable in a frame $f$ the semantic analysis phase calls F_allocLocal ( $f$, TRUE) where the boolean argument specifies whether the local variable escapes.
- Calls to allocLocal need not all come immediately after the frame is created. For example,

```
function f() =
let var v := 6
    in print(v); let var v := 7 in print(v) end;
        print(v); let var v := 8 in print(v) end;
        print(v)
end
```

In this example, the second and third $v$ variables could be held in the same temporary location. A clever compiler might optimise the size of the frame to make use of this fact.

## Calculating escapes

- Local variables that do not escape can be allocated in registers. Escaping variables must be allocated in the frame. A FindEscape function can look for escaping variables and record this information in the abstract syntax tree for the program. This processing must happen before semantic analysis begins.
- The traversal function for FindEscape will be a mutual recursion on abstract syntax exp's and var's, just like the type-checker.
- Again, like the type checker it will build up an environment which maps variables to bindings. Here the bindings simply record if the variable escapes.


## Temporaries and labels

- The compiler's semantic analysis phase will choose registers for parameters and local variables, and also to choose machine-code addresses for procedure bodies. But it is too early to determine exactly which registers are available or exactly where a procedure body will be located. We use the word temporary to mean a value that is temporarily held in a register, and the word label to mean some machine-language location whose exact address is yet to be determined.
- The Temp module provides the following functions.
- Temp_newtemp() return a new temporary
- Temp_newlabel() return a new label
- Temp_namedlabel (string) returns a new label whose assembly language name is string.


## Constructors for temporaries and labels

```
Temp_temp Temp_newtemp(void);
Temp_tempList Temp_TempList(Temp_temp h, Temp_tempList t);
Temp_label Temp_newlabel(void);
Temp_label Temp_namedlabel(string name);
string Temp_labelstring(Temp_label s);
Temp_labelList Temp_LabelList(Temp_label h, Temp_labelList t);
Temp_map Temp_empty(void);
Temp_map Temp_layerMap(Temp_map over, Temp_map under);
void Temp_enter(Temp_map m, Temp_temp t, string s);
string Temp_look(Temp_map m, Temp_temp t);
Temp_map Temp_name(void);
```


## Environment entries revisited

In the semantic analysis phase of the Tiger compiler, transDec creates a new "nesting level" for each function body. The nesting level is stored in the FunEntry data structure for the function. The FunEntry also needs the label of the function's machine code entry point.

```
/* New versions of VarEntry and FunEntry */
struct E_enventry_ {
    enum { E_varEntry, E_funEntry } kind;
    union { struct { Tr_access access; Ty_ty ty; } var;
        struct { Tr_level level; Temp_label label;
            Ty_tylist formals; Ty_ty result; } fun;
        } u;
    };
```

- The Translate module knows that each frame contains a static link. The static link is passed in a register and stored in the frame. Since the static link behaves so much like a formal parameter we treat it as one, specifically one which escapes.
- Then, just before creating a new frame we add another formal parameter to the front for the static link.
F_newFrame(label, U_BoolList(TRUE, fmls))

What comes back is an F_frame. In this frame are three offset values, accessible by calling F_formals(frame).

## Translation to intermediate code

- The semantic analysis phase of a compiler must translate abstract syntax into machine code. It can do this at the same time as typechecking, or after.
- Although it would be possible to translate directly to real machine code this would hinder portability and modularity. An intermediate representation (IR) is an abstract machine language which can express target machine operations without too much machinespecific detail. It is independent of the details of the source language.
- Thus the front end of a compiler does lexical analysis, parsing, semantic analysis, and translation to an intermediate representation. The back end does optimisation of the intermediate representation and translation to machine language.


## Statements in the IR language

MOVE (TEMP $t, e$ ) Evaluate $e$ and move it to temporary $t$
MOVE (MEM $e_{-} 1, e_{\_} 2$ ) Evaluate $e_{-} 1$, yielding address $a$. Then evaluate $e \_2$ and store the result into wordSize bytes of memory starting at $a$.

EXP (e) Evaluate $e$ and discard the result.
JUMP ( $e, l a b s$ ) Transfer control to address $e$. The list of labels specifies all possible locations that $e$ can evaluate to, for dataflow analysis.
CJUMP ( $o, e_{\_} 1, e_{\_} 2, t, f$ ) Evaluate $e_{\_} 1$, then $e_{\_} 2$. Compare the results using the relational operator $o$. If the result is true, jump to $t$; otherwise jump to $f$. The relational operators are EQ, NE, LT, GT, LE, GE, ULT, UGT, ULE, UGE.

SEQ ( $s \_1, s_{\_} 2$ ) The statement $s_{-} 1$ followed by $s \_2$.
LABEL $(n)$ Define the constant value of name $n$ to be the current machine code address.

## Kinds of expressions

- The Tiger language contains expressions of different kinds: some produce values and some do not (assignments, loops and procedure calls). Expressions with Boolean values can be best represented by a conditional jump.
- We will make a union type, as usual, with a kind tag. This will model three kinds of expressions.

Ex: This is a value-producing expression, represented as a Tr_exp.
Nx: An expression with no result, represented as a Tr_stm.
Cx: A conditional represented as a statement which may jump to a true-label or a false-label. These labels have yet to be filled in. The statement never "falls through".

- It must be convenient for the semantic analysis phase to produce.
- It must be convenient to translate into real machine language, for all the desired target machines.
- Each construct must have a clear and simple meaning, so that optimising transformations that rewrite the intermediate representation can easily be specified and implemented.

The intermediate representation should have individual components which are extremely simple: a single fetch, store, add, move or jump. A piece of abstract syntax can be translated into a set of abstract machine instructions and groups of abstract machine instructions can be brought together to form genuine machine instructions.

## Expressions in the IR language

CONST ( $i$ ) The integer constant $i$.
NAME ( $n$ ) The symbolic constant $n$, corresponding to an assembly language label.

TEMP $(t)$ Temporary $t$. A temporary in the abstract machine is similar to a register in the real one.
BINOP ( $o, e_{\_} 1, e_{\_} 2$ ) The application of operator $o$ to $e_{-} 1$ and $e_{-} 2$, evaluated in that order. The operators are PLUS, MINUS, MUL, DIV, AND, OR, XOR, LSHIFT, RSHIFT, ARSHIFT.

MEM (e) The contents of wordSize bytes of memory from address $e$.
CALL $(f, l)$ The application of function $f$ to argument list $l$, evaluated in that order.
$\operatorname{ESEQ}(s, e)$ The statement $s$ is evaluated for its side effect then $e$ for its result.

## Implementing expressions

```
* in translate.h */
typedef struct Tr_exp_ *Tr_exp
/* in translate.c */
struct CX { patchList trues; patchList falses; T_stm stm; };
struct Tr_exp_
    { enum { Tr_ex, Tr_nx, Tr_cx } kind;
        union { T_exp ex; T_stm nx; struct Cx cx; } u;
    };
static Tr_exp Tr_Ex(T_exp ex);
static Tr_exp Tr_Nx(T_stm nx);
static Tr_exp Tr_Cx(patchList trues, patchList falses, T_stm stm);
```


## Translating an expression

The Tiger expression $\mathrm{a}>\mathrm{b} \mid \mathrm{c}<\mathrm{d}$ might translate to the following.

```
Temp_label z = Temp_newlabel();
T_stm s1 =
    T_Seq(T_Cjump(T_gt, a, b, NULL}t, z)
            T_Seq(T_Label(z),
                T_Cjump(T_lt, c, d, NULL}t,\mp@subsup{N}{NULL}{f}))\mathrm{ )
```

The destinations $t$ and $f$ are not known yet, although z is. The statement contains NULL Nand $_{t}$, which are yet to be filled in.

```
void doPatch(patchList tList, Temp_label label) {
    for (; tList; tList=tList->tail)
        *(tList->head) = label;
}
By calling joinPatch(first, second) we link two patch lists together.
patchList joinPatch(patchList first, patchList second) {
    if (!first) return second;
    /* go to end of list */
    for (; first->tail; first=first->tail);
    first->tail = second;
    return first;
}
```

The unEx conversion function

```
static T_ exp unEx(Tr_exp e) {
    switch(e->kind) {
        case Tr_ex:
        return e->u.ex;
        case Tr_cx: {
            Temp_temp r = Temp_newtemp();
            Temp_label t = Temp_newlabel(), f = Temp_newlabel();
            doPatch(e->u.cx.trues, t);
            doPatch(e->u.cx.falses, f),
            return T_Eseq(T_Move(T_Temp(r), T_Const(1)),
                    T_Eseq(e->u.cx.stm,
                    T_Eseq(T_Label(f),
                        T_Eseq(T_Move(T_Temp(r), T_Const(0)),
                        T_Eseq(T_Label(t),
                                    T_Temp(r))))));
    }
    case Tr_nx:
        return T_Eseq(e->u.nx, T_Const(0));
    }
    assert(0); /* can't get here */
}
```


## Extending the Frame module

The Frame module holds all machine-dependent definitions; here we add a frame-pointer register FP and a constant whose value is the machine's natural word size.

```
/* frame.h */
    \vdots
Temp_temp F_FP(void);
extern const int F_wordSize;
T_exp F_Exp(F_access acc, T_exp framePtr);
```


## Using the Frame module

- The function F_Exp is used by Translate to turn an F_access into a Tree expression. The framePtr argument is the address of the stack frame that the F_access is in.
- Thus for an access $a$ such as InFrame $k$, we have

$$
F_{-} \operatorname{Exp}\left(a, T_{-} T e m p\left(F_{-} F P()\right)\right)
$$

returns the result
MEM(BINOP(PLUS, TEMP fp , CONST $k)$ )
variables or formal parameters. A function might access a variable which was declared by its parent. When a variable is declared at an outer level of static scope like this we access it by following static links. in general, we have this:

```
MEM(BINOP(PLUS, CONST }\mp@subsup{k}{n}{}\mathrm{ ,
    MEM(BINOP(PLUS, CONST }\mp@subsup{k}{n-1}{
    MEM(BINOP(PLUS, CONST }\mp@subsup{k}{1}{
    TEMP fp))}\cdots\mp@code{)})\mathrm{ )
```

- The constants $k_{1}, \ldots, k_{n-1}$ are the various static link offsets in nested functions and $k_{n}$ is the offset of $x$ in its own frame.


## Translating complex logical expressions

- The result of a comparison operator will be a Cx expression; a statement $s$ which will jump to any true-destination and falsedestination which are specified.
- Making a Cx expression from a single use of the comparison operators of the Tiger language is relatively straightforward. For example, the expression $\mathrm{x}<5$ will be translated as a Cx with

$$
\begin{aligned}
\text { trues } & =\{t\} \\
\text { falses } & =\{f\} \\
\text { stm } & \left.=\text { CJUMP }^{(L T}, x, \text { CONST } 5, \text { NULL }_{t}, \text { NULL }_{f}\right)
\end{aligned}
$$

- However, Cx expressions can themselves form part of a larger truth-valued expression. The \& (and) and । (or) operators of the Tiger language combine logical expressions using short-circuit evaluation. These have already been translated into if-expressions in the abstract syntax.
is to treat $e_{1}$ as a Cx expression, and $e_{2}$ and $e_{3}$ as Ex expressions. We need to fit the three sub-expressions together into a stream of statements into which we can insert labels for the then branch and the else branch.
- We proceed by applying unCx to $e_{1}$ and unEx to $e_{2}$ and $e_{3}$. We make two labels to which the conditional will branch, $t$ and $f$. Allocate a temporary r , and after label t move $e_{2}$ to r ; after label f move $e_{3}$ to $r$. Both branches should finish by jumping to a newly created "join" label.


## Problem cases for conditional expressions (1)

Following our proposed translation would lead to inefficient code. A common use of an if-expression is to allow the selection between two "statements" (expressions which produce no value). In this case their representation will have been constructed using the Nx constructor. Applying unEx to them will work but it might be better to recognise this case specially.

## Problem cases for conditional expressions (2)

Worse, if $e_{2}$ or $e_{3}$ is a Cx expression then the use of unEx produces even longer statement sequences. Again, we should recognise this case specially. For example, an if-expression such as the following
if ( $\underbrace{\text { if } x<5 \text { then } a>b \text { else } 0}-x<5 \& a>b$ ) then $\ldots$
should be translated as

$$
\operatorname{SEQ}\left(s_{1}(z, f), \operatorname{SEQ}\left(\operatorname{LABEL} z, s_{2}(t, f)\right)\right)
$$

for some new label $z$. The notation $s_{1}(z, f)$ denotes the Cx statement $s_{1}$ with its trues labels filled in with $z$ and its f alses labels filled in with $f$.

## Record and array creation

- The Tiger language expression $t\left\{f_{1}=e_{1}, f_{2}=e_{2}, \ldots, f_{n}=e_{n}\right\}$ creates an $n$-element record of type $t$ with fields $f_{i}$ initialised to the values of expressions $e_{i}$. Such a record may outlive the instance of the function which created it, so it cannot be allocated on the stack. Instead it must be allocated in another memory area called the heap. Heap-allocated values must be reclaimed by a garbage collector when an analysis determines that they are no longer reachable.
- To create a record allocate an $n$-word memory area into a new temporary $r$ (using malloc). A series of MOVE instructions can initialise offsets $0, W, 2 W, \ldots,(n-1) W$ from $r$. The result of the whole expression is TEMP $r$.
- Array creation is very similar except that all locations are initialised to the same value.


## Calling runtime-system functions

- To call an external function named initArray with arguments $a$ and $b$, simply generate a call such as

$$
\begin{gathered}
\text { CALL (NAME(Temp_namedlabel("initArray")), } \\
\text { T_ExpList }(a, \text { T_ExpList }(b, \text { NULL })))
\end{gathered}
$$

This refers to an external function written in C or assembler.

- However, on some operating systems the C compiler puts an underscore at the beginning of each label; the calling conventions of C are different from those of Tiger functions; C functions don't expect to receive a static link, and so on. We encapsulate all of this in the Frame module, as usual.

```
/* frame.h */
!
T_exp F_externalCall(string s, T_expList args);
```

done:

If a break statement occurs within the body (and not nested within an interior while loop or a procedure body) then the translation is simply a jump to the label done.

- So that transExp can translate break statements, it will have a new formal parameter break that is the done label of the nearest enclosing loop.


## For loops

A for statement can be expressed as a combination of let .. in .. end and a while loop.

$$
\text { for } i:=l o \text { to } h i
$$

do body

$$
\begin{aligned}
& \text { let var } i:=l o \\
& \quad \text { var limit }:=\text { hi } \\
& \text { in while } i<=\text { limit } \\
& \quad \text { do }(\text { body } ; i:=i+1) \\
& \text { end }
\end{aligned}
$$

This is almost right. It fails when hi evaluates to the maximum representable integer. The final increment in the loop body will either overflow or wrap around. Neither behaviour is correct. A production quality compiler would generate an extra test for this.

## Function calls

Translating a function call $f\left(e_{-} 1, \ldots, e_{-} n\right)$ is relatively simple except that the static link must be added as an extra argument.

$$
\text { CALL(NAME } \left.l_{-} f,\left[s l, e_{-} 1, \ldots, e_{-} n\right]\right)
$$

Here $l_{-} f$ is the label for $f, s l$ is the static link and the $e_{-} i$ are the translations of the argument list $a \_i$. To calculate the static link both

## Variable definition

- The transDec function which we saw when considering typechecking updates the value environment and type environment. These are used in processing the body of a let expression.
- In Tiger, variables are always given their initial value at the time that they are declared and so a variable declaration must also produce a Tree expression that must be put just before the body of the let. Therefore transDec must also return a Tr _exp containing assignment expressions which accomplish these initialisations.
- If transDec is applied to function and type declarations, the Tr_exp must be a "no-op" expression such as Tr_Ex (T_Const (0)).


## Function definition

Each Tiger function is translated into a prologue (1-5), a body (6) and an epilogue (7-11).

1. pseudo-instructions for the function beginning
2. a label definition for the function name
3. an instruction to adjust the stack pointer
4. instructions to save "escaping" arguments
5. store instructions to save any callee-save registers
6. the function body
7. an instruction to deliver the function result
8. load instructions to restore the callee-save registers
9. an instruction to reset the stack pointer
10. a jump to the return address
11. pseudo-instructions for the function end

## Frames and function definitions

- Some of the parts of the translation of a function definition depend on knowledge of the frame size. In order to keep this information separated from the machine-independent parts of the compiler we find that we should add more definitions to the Frame module.
- For return values we need a distinguished return value location.

```
Temp_temp F_RV(void);
```

- Moving formal parameters and the saving and restoring of calleesave registers should also be handled by a function in the Frame module.

T_stm F_procEntryExit1(F_frame frame, T_stm stm);

## Translating the Tree language to machine-language

Some features of the Tree language must be adapted before translation into machine language.

- The CJump instruction can jump to either of two labels but real machine's conditional jumps fall through if the condition is false.
- ESEQ nodes within expressions are inconvenient because they make different orders of evaluating subtrees yield different results.
- CALL nodes within expressions cause the same problem.
- CALL nodes within the argument-expressions of other CALL nodes cause problems when sharing formal parameter registers.


## Rewriting trees

- We can take any tree and rewrite it into an equivalent tree without the problems due to ESEQ and CALL nodes. Without these cases we will have a simple collection of SEQ nodes clustered together at the top of the tree. This can easily be turned into a linear list of instructions.
- The transformation is done in three stages:

1. A tree is rewritten into a list of canonical trees without SEQ or ESEQ nodes.
2. This list is grouped into basic blocks, which contain no internal jumps or labels.
3. The basic blocks are ordered into a set of traces in which every CJUMP is immediately followed by its false label.

## Canonicalisation module

The Canon module has these tree rearrangement functions.

```
/* canon.h */
    \vdots
    typedef struct C_stmListList_ *C_stmListList;
    struct C_block { C_stmListList stmLists; Temp_label label; };
    struct C_stmListList_ { T_stmList head; C_stmListList tail; };
    T_stmList C_linearize(T_stm stm);
    struct C_block C_basicBlocks(T_stmList stmList);
    T_stmList C_traceSchedule(struct C_block b);
```

Linearize removes the ESEQs and moves the CALLs to top level. BasicBlocks groups statements into sequences of straight-line code. TraceSchedule orders the blocks so that each CJUMP is followed by its false label.

1. There is no use of the SEQ or ESEQ nodes in the tree
2. The parent of each CALL is either of the form $\operatorname{EXP}(\ldots)$ or of the form MOVE(TEMP $t, \ldots$ ). The former correspond to procedure calls (which do not return a result) and the latter to function calls (which do).

- ESEQ nodes are eliminated by moving them further and further up the tree until they become SEQ nodes. Many useful transformations on trees can be defined. These can be repeatedly applied to bring about the alterations which we want to make.


## Commutativity

We wish to re-arrange intermediate representation trees so that all of the ESEQ nodes move up the tree. When we consider a subtree produced by $z^{+}(\mathrm{x}:=\mathrm{y}, \mathrm{x})$ such as
$\operatorname{BINOP}(\operatorname{PLUS}, \operatorname{MEM}(z), \operatorname{ESEQ}(\operatorname{MOVE}(\operatorname{MEM}(x), y), \operatorname{MEM}(x)))$
with a subtree such as
$\operatorname{ESEQ}(\operatorname{MOVE}(\operatorname{MEM}(x), y), \operatorname{BINOP}(\operatorname{PLUS}, \operatorname{MEM}(z), \operatorname{MEM}(x)))$
as would have been produced by ( $x:=y, z+x$ ) we need to determine whether the $\mathrm{x}:=\mathrm{y}$ and the z commute.

We cannot always determine this at compile time. Under some runtime evaluations x and z might be aliases. That is, they might be two different names for the same location. A situation such as this can be created by parameter passing where variables may be passed by reference.

## General rewriting rules

Given a Tree statement or expression we first identify the subexpressions and extract these, as directed by our tree transformations. We then rebuild our statement or expression with the extracted subexpressions replaced in the right way.

```
typedef struct expRefList_ *expRefList;
struct expRefList_ { T_exp *head; expRefList tail; };
struct stmExp { T_stm s; T_exp e; };
/* Combine the statement parts from ESEQs */
static T_stm reorder(expRefList rlist);
static struct stmExp do_exp(T_exp exp);
static T_stm do_stm(T_stm stm);
```


## Processing expressions

```
static struct stmExp do_exp(T_exp exp) {
switch (exp->kind) {
    case T_BINOP:
        return StmExp(
        reorder(ExpRefList(&exp->u.BINOP.left,
            ExpRefList(&exp->u.BINOP.right,NULL))), exp); /* MEM(e) */
    case T MEM:
        return StmExp(
        reorder(ExpRefList(&exp->u.MEM,NULL)), exp); /* ESEQ(s, e) */
        struct stmExp x = do_exp(exp->u.ESEQ.exp);
        return StmExp(seq(do_stm(exp->u.ESEQ.stm), x.s), x.e)
        }
    }
    case T_CALL: /* CALL(f, l) */
        return StmExp(reorder(get_call_rlist(exp)), exp);
    default: /* CONST, NAME and TEMP */
        return StmExp(reorder(NULL), exp);
}}
```

```
function eval () : int =
let
    type recvar = { val : int }
    var y: int := 10
    var z: recvar := recvar{ val = 0 }
    function alias (x: recvar) : int =
            (x.val := y, z.val + x.val)
in
    alias (z)
end
```

In C++ reference declarations give rise to aliasing. The example here could be coded as int alias (int\& $x$ ) \{return ( $x=y, z+x$ ); \}.

## Conservatively estimating commutativity

- Although we cannot guarantee to always determine the validity of commuting a statement with an expression at compile-time we can at least conservatively approximate commutativity. This means that we say either "they definitely do commute" or "perhaps they don't commute".
- For example, we know that any statement will commute with any constant value (say, the expression $\operatorname{CONST}(n)$ ) so we can always justify special cases such as the following.

$$
\begin{aligned}
& \operatorname{BinOP}(o p, \operatorname{Const}(n), \operatorname{ESEQ}(s, e)) \\
= & \operatorname{ESEQ}(s, \operatorname{BinOP}(o p, \operatorname{Const}(n), e))
\end{aligned}
$$

## Eliminating No-Ops

In the case that one of two adjacent statements is a No-Op we can omit it from the emitted instructions.

```
/* test for no-ops */
static bool isNop(T_stm x) {
    return x->kind == T_EXP && x->u.EXP->kind == T_CONST;
}
/* combine statements, omitting no-ops */
static T_stm seq(T_stm x, T_stm y) {
    if (isNop(x)) return y;
    if (isNop(y)) return x;
    return T_Seq(x,y);
}
```


## Processing statements

```
static T_stm do_stm(T_stm stm) {
    switch (stm->kind) {
    case T_SEQ:
        return seq(do_stm(stm->u.SEQ.left), do_stm(stm->u.SEQ.right));
    case T_JUMP:
        return seq(reorder(ExpRefList(&stm->u.JUMP.exp, NULL)), stm);
    case T_CJUMP:
    return seq(reorder(ExpRefList(&stm->u.CJUMP.left
                                    ExpRefList(&stm->u.CJUMP.right,NULL))), stm);
    case T_MOVE: ... to do ..
    case T_EXP:
    if (stm->u.EXP->kind == T_CALL)
                return seq(reorder(get_call_rlist(stm->u.EXP)), stm);
        else return seq(reorder(ExpRefList(&stm->u.EXP, NULL)), stm);
default:
        return stm;
}}
```

return seq(reorder(get_call_rlist(stm->u.MOVE.src)), stm);
else if (stm->u.MOVE.dst->kind $==$ T_TEMP)
return seq(reorder(ExpRefList(\&stm->u.MOVE.src, NULL)), stm); else if (stm->u.MOVE.dst->kind == T_MEM)
return seq(reorder(ExpRefList(\&stm->u.MOVE.dst->u.MEM, ExpRefList(\&stm->u.MOVE.src, NULL))), stm);
else if (stm->u.MOVE.dst->kind == T_ESEQ) \{
T_stm s = stm->u.MOVE.dst>u.ESEQ.stm;
stm->u.MOVE.dst = stm->u.MOVE.dst->u.ESEQ.exp;
return do_stm(T_Seq(s, stm));
\}
assert(0); /* dst should be temp or mem only */ . other cases as before ...
\}\}

## Commuting statements and expressions

With the assistance of do_exp and do_stm, the reorder function can pull the statement $s_{-} i$ out of each expression $e_{-} i$ on its list of references, going from right to left. Temporaries are introduced if the statement and expression do not commute. For example, there are three possible re-arrangements of $\left[e_{-} 1, e_{-} 2, \operatorname{ESEQ}\left(s, e_{-} 3\right)\right]$ :

- $s ;\left[e_{-} 1, e_{-} 2, e_{-} 3\right]$
- $\operatorname{SEQ}\left(\operatorname{MOVE}\left(t_{-} 1, e_{-} 1\right), s\right) ;\left[\operatorname{TEMP}\left(t_{-} 1\right), e_{-} 2, e_{-} 3\right]$
- $\operatorname{SEQ}\left(\operatorname{MOVE}\left(t_{-} 1, e_{-} 1\right), \operatorname{SEQ}\left(\operatorname{MOVE}\left(t_{-} 2, e_{-} 2\right), s\right)\right) ;\left[\operatorname{TEMP}\left(t_{-} 1\right), \operatorname{TEMP}\left(t_{-} 2\right), e_{-} 3\right]$

The following function estimates whether two expressions commute.

```
static bool commute(T_stm x, T_exp y) {
    return isNop(x) || y->kind==T_NAME || y->kind==T_CONST;
}
```

then the second call will overwrite the RV register before the PLUS can be executed. As always, we can solve this by introducing a new temporary.

$$
\operatorname{CALL}(f u n, \text { args }) \rightarrow
$$

$$
\operatorname{ESEQ}(\operatorname{MOVE}(\operatorname{TEMP} t, \operatorname{CALL}(f u n, \text { args })), \operatorname{TEMP} t)
$$

## Linearising statements

Once an entire function body $s_{-} 0$ is processed with do_stm, the result is a tree $s_{-} 0^{\prime}$ where all the SEQ nodes are near the top. The linearize function repeatedly applies $\operatorname{SEQ}(\operatorname{SEQ}(a, b), c)=\operatorname{SEQ}(a, \operatorname{seq}(b, c))$.

```
static T_stmList linear(T_stm stm, T_stmList right) {
    if (stm->kind == T_SEQ)
        return linear(stm->u.SEQ.left,
                    linear(stm->u.SEQ.right,
                    right));
    else return T_StmList(stm, right);
}
T_stmList C_linearize(T_stm stm) {
    return linear(do_stm(stm), NULL);
}
```


## Compiling conditional branches

- Another aspect of the Tree language which must be compiled down into simpler instructions is the two-way branch of the CJUMP operator. On a real machine the conditional jump either transfers control (on true) or "falls through" to the next instruction.
- To make the trees easy to translate into machine instructions we rearrange them so that every CJUMP (cond, $\left.l_{-} t, l_{-} f\right)$ is immediately followed by LABEL ( $\left.l_{-} f\right)$, its "false branch".
- We make this transformation in two stages: we first take the list of canonical trees and form them into basic blocks; then we order the basic blocks into a trace.


## Basic blocks

- In determining where the jumps go in a program we are analyzing the program's control flow. We consider only the sequencing of instructions, independent from the program's data. In this setting we cannot say whether the condition of a jump will evaluate to true or false and so we consider both possibilities.
- A basic block is a sequence of statements which is entered at the beginning and left at the end. That is:

[^0]
## Finding basic blocks

We divide a long sequence of statements into basic blocks as follows.

- The sequence is scanned from beginning to end.
- Whenever a LABEL is found, a new block is started.
- Whenever a JUMP or CJUMP is found, a block is ended.
- If this leaves any block not ending with a JUMP or CJUMP then a JUMP to the next block's label is added to the end of the block.
- If this leaves any block not beginning with a LABEL then a new label is invented and put there.


## Traces

- Basic blocks can be arranged in any order and the result of executing the program will be the same. We take advantage of this fact in two ways:
- we can move the false-labelled block immediately after its conditional branch; and
- we can arrange that many of the unconditional jumps are followed by their target label.
- A trace is a sequence of statements which could be consecutively executed during the execution of the program. For our purposes we want to make a set of traces that exactly covers the program: each block must be in exactly one trace. To minimise the number of JUMPs from one trace to another, we would like to have as few traces as possible in our covering set.

Start a new (empty) trace, call it $T$.
Remove the head element $b$ from $Q$.
while $b$ is not marked \{
Mark $b$; append $b$ to the end of $T$.
Examine the successors of $b$ (blocks to which $b$ branches)
if there is any unmarked successor $c$
$b \leftarrow c$
\}
/* All the successors of $b$ are marked. */
End the current trace $T$.
\}

## Instruction selection

- The Tree language expresses only one operation in each tree node: memory fetch or store, addition or subtraction, conditional jump, and so on. A real machine instruction can often perform several of these in the same instruction.
- Finding the appropriate machine instructions to implement a given intermediate representation tree is the job of the instruction selection phase of a compiler.
- We express a machine instruction as part of an IR tree, called a tree pattern. Then instruction selection becomes the task of tiling (or covering) the tree with a minimal set of non-overlapping tree patterns.
language expression using instructions from the Jouette instruction set.

$$
\mathrm{a}[\mathrm{i}]:=\mathrm{x}
$$

- We use the following facts about the variables in the expression and the Jouette architecture.
- The variable i is a register variable.
- The variables a and x are frame resident. (An array variable is really a pointer to an array.)
- The wordsize $W$ is 4 (bytes). Thus a [i] is represented by

$$
\operatorname{MEM}(\operatorname{BINOP}(+, a, \operatorname{BINOP}(*, i, 4)))
$$

- Register $r_{0}$ always contains zero.


## Optimal and optimum tilings

- The best tiling of a tree corresponds to an instruction sequence of least cost. If we assign a cost to each instruction then we could define an optimum tiling as one whose tiles sum to the lowest possible value.
- In contrast an optimal tiling is one where no two adjacent tiles can be combined into a single tile of lower cost.
- If there is some tree pattern that can be split into several tiles of lower combined cost then we should remove that pattern from our catalogue of tiles before we begin.
- Every optimum tiling is also optimal, but not vice versa.


## The "Maximal Munch" algorithm

- The tiles which are used to tile the IR tree can be graded in terms of the number of their nodes. The tile for ADD has one node, the tile for SUBI has two and the tile for MOVEM has three. Loosely speaking, we can say that MOVEM is 'larger' than SUBI and that SUBI is 'larger' than ADD.
- The algorithm for optimal tiling is called Maximal Munch and proceeds in this way.
- Starting at the root of the tree, find the largest tile that fits. (If two tiles of equal size match at the root then choose either one.)
- Cover the root node (and others near it) with this tile, leaving several subtrees.
- Repeat the algorithm for each subtree.


## Dynamic programming

- Maximal Munch always finds an optimal tiling, but not necessarily an optimum one. A dynamic programming algorithm can find the optimum.
- The algorithm for optimum tiling proceeds in this way.
- Work bottom-up by first finding the costs of all the children (and grandchildren, ...) of node $n$. Then each tile is matched against node $n$.
- A tile has zero or more leaves where subtrees can be attached. For each tile $t$ of cost $c$ that matches at node $n$ there will be subtrees $s_{1}, \ldots s_{n}$ of cost $c_{1}, \ldots, c_{n}$. So the cost of matching tile $t$ is $c+c_{1}+\cdots+c_{n}$.
- Of all the tiles $t_{j}$ which match at node $n$, the one with the minimum cost match is chosen and the (minimum) cost of node $n$ is thus computed.


## Instruction emission

- Once the cost of the root node (and thus the entire tree) is found, the instruction emission phase begins. The algorithm is as follows:
Emission (node $n$ ): for each of the leaves $l_{i}$ of the tile selected at node $n$, perform Emission $\left(l_{i}\right)$. Then emit the instruction matched at node $n$.
- Note that $\operatorname{Emission}(n)$ does not apply itself recursively to the children of node $n$, but to the leaves of the tile which matched.
For example, for MEM ( + , CONST 1, CONST 2) we emit instructions for CONST 1 and for the node itself but not for any tile rooted at the + node.


## Liveness analysis

- The front end of a compiler translates programs into an intermediate language with an unlimited number of temporary storage locations (temporaries). The translated program must ultimately run on a machine with a fixed number of registers.
- Even if their scopes overlap, two temporaries can fit into a single register if they are never in use at the same time. Following from this, many temporaries can fit into few registers; if they don't all fit, the excess temporaries can be kept in memory.
- A compiler needs to analyse the intermediate representation tree to determine which temporaries are in use at the same time. A variable is said to be live if it holds a value which may be needed in the future. This analysis is called liveness analysis.
control-flow graph. Each statement in the program is a node in the graph. If statement $S_{1}$ can be followed by statement $S_{2}$ then there is an edge from $S_{1}$ to $S_{2}$.
- In determining the live range of a variable we work backwards through the control-flow graph, examining statements.
- If a variable $b$ is used in a statement (i.e. its value is taken) then $b$ must have been live on the edge leading to the statement.
- If a variable is assigned into in a statement (i.e. its value is set) then it must be dead on the edge leading to the statement.


## Utilising the results

The effect of understanding and making use of the results of liveness analysis is to allow us to see that two variables could be replaced by one.

|  | $a \leftarrow 0$ |  | $a \leftarrow 0$ |
| :--- | :--- | :--- | :--- |
| $L_{1}:$ | $b \leftarrow a+1$ | $L_{1}:$ | $a \leftarrow a+1$ |
|  | $c \leftarrow c+b$ | $c \leftarrow c+a$ | $L_{1}:$ |
|  | $a \leftarrow b \leftarrow 0$ | $b \leftarrow b+1$ |  |
|  |  |  | $c \leftarrow c+b$ |
|  | if $a<N$ goto $L_{1}$ |  | $a \leftarrow a * 2$ |
|  | if $a<N$ goto $L_{1}$ |  | $b \leftarrow b * 2$ |
|  | return $c$ | return $c$ |  |
|  |  |  | return $c$ |

edges which come from predecessor nodes. The set Pred $[n]$ is all

- An assigment to a variable or temporary defines that variable. An occurrence of a variable in an expression uses the variable. We can speak of the def of a variable as the set of node that use it, and conversely for a graph node. Similarly we can speak of the uses of a variable or graph node.
- A variable is live on an edge if there is a directed path from that edge to a use of the variable which does not go through any defs. A variable is live-in if it is live on any of the in-edges of a node. It is live-out if live on any out-edges.


## Calculation of liveness by iteration

```
/ * initialise the edge sets * /
for each \(n\)
    in \([n] \leftarrow\} ; \quad\) out \([n] \leftarrow\} ;\)
```

repeat
for each $n$
/ * record the old values * /
in $^{\prime}[n] \leftarrow$ in $[n] ; \quad$ out $^{\prime}[n] \leftarrow$ out $[n] ;$
/ * compute the new values */
in $[n] \leftarrow$ use $[n] \cup($ out $[n]-\operatorname{def}[n]) ;$
out $[n] \leftarrow\}$;
for each $s$ in succ $[n]$
out $[n] \leftarrow$ out $[n] \cup$ in $[s] ;$
until $\mathrm{in}^{\prime}[n]=$ in $[n]$ and out ${ }^{\prime}[n]=$ out $[n]$ for all $n$

INDEX INDEX

## Index



| Left-to-right parse, leftmost derivation, 1 -symbol lookahead, 12 | stack frame, 37 <br> stack pointer, 37 |
| :---: | :---: |
| Left-to-right parse, rightmost derivation, k-token lookahead, 14 | start symbol, 7 <br> states, 16 |
| linear, 70 | static link, 42 |
| live, 76 | static nesting depth, 42 |
| live range, 77 | stmExp, 67 |
| live-in, 78 | strings, 6 |
| live-out, 78 | successor, 78 |
| liveness analysis, 76 | symbol tables, 27 |
| lookahead, 14 | symbols, 6 |
| Maximal Munch, 75 | Temp_empty, 48 |
|  | Temp_enter, 48 |
| non-terminal, 7 | Temp_LabelList, 48 |
| non-terminals, 11 | Temp_labelstring, 48 |
| optimal, 74 | Temp_layerMap, 48 |
| optimum, 74 | Temp_look, 48 |
| out-edges, 78 | Temp_name, 48 |
|  | Temp_namedlabel, 48 |
| parse tree, 24 | Temp_newlabel, 48 |
| PatchList, 53 | Temp_newtemp, 48 |
| patchList, 53 | Temp_TempList, 48 |
| patchList_, 53 | temporaries, 76 |
| position stack, 25 | temporary, 47 |
| precedence, 9 | terminal, 7 |
| predecessor, 78 | terminals, 11 |
| predictive parsing table, 12 | terms, 9 |
| productions, 6 | tiling, 73 |
| prologue, 62 | Tr_Cx, 52 |
|  | Tr_Ex, 52 |
| recursive descent, 9 | Tr_exp, 52 |
| reduce-reduce, 21 | Tr_exp_, 52 |
| reference declarations, 66 | Tr_Nx, 52 |
| register allocator, 39 | trace, 71, 72 |
| register windows, 40 | traces, 64 |
| registers, 38 | transExp, 32, 33 |
| reorder, 67 | transition table, 15 |
| return address, 41 | transVar, 33, 34 |
| rewrite, 64 | tree pattern, 73 |
|  | type declaration, 30 |
| scope, 27 | type environment, 31 |
| semantic analysis, 27 | type expression, 30 |
| semantic value, 23,25 | uncx, 53 |
| seq, 68 | unEx, 53 |
| shift-reduce, 21 | unNx, 53 |
| stack, 14 | uses, 78 |


[^0]:    - the first statement is a LABEL;
    - the last statement is a JUMP or CJUMP; and
    - there are no other LABELS, JUMPS or CJUMPS.

