Parsing IV
Bottom-up Parsing

COMP 412
Fall 2005

Parsing Techniques

Top-down parsers *(LL(1), recursive descent)*

- Start at the root of the parse tree and grow toward leaves
- Pick a production & try to match the input
- Bad "pick" $\Rightarrow$ may need to backtrack
- Some grammars are backtrack-free *(predictive parsing)*

Bottom-up parsers *(LR(1), operator precedence)*

- Start at the leaves and grow toward root
- As input is consumed, encode possibilities in an internal state
- Start in a state valid for legal first tokens
- Bottom-up parsers handle a large class of grammars

Bottom-up Parsing (definitions)

The point of parsing is to construct a ***derivation***

A derivation consists of a series of rewrite steps

$$S \Rightarrow \gamma_0 \Rightarrow \gamma_1 \Rightarrow \gamma_2 \Rightarrow \ldots \Rightarrow \gamma_{n-1} \Rightarrow \gamma_n \Rightarrow \textit{sentence}$$

- Each γ_i is a sentential form
  - If γ contains only terminal symbols, γ is a sentence in $L(G)$
  - If γ contains 1 or more non-terminals, γ is a sentential form
- To get γ_i from γ_{i-1}, expand some NT $A \in \gamma_{i-1}$ by using $A \rightarrow \beta$
  - Replace the occurrence of $A \in \gamma_{i-1}$ with β to get γ_i
  - In a leftmost derivation, it would be the first NT $A \in \gamma_{i-1}$

A *left-sentential form* occurs in a *leftmost* derivation

A *right-sentential form* occurs in a *rightmost* derivation

Bottom-up Parsing

A bottom-up parser builds a derivation by working from the input sentence back toward the start symbol S

$$S \Rightarrow \gamma_0 \Rightarrow \gamma_1 \Rightarrow \gamma_2 \Rightarrow \ldots \Rightarrow \gamma_{n-1} \Rightarrow \gamma_n \Rightarrow sentence$$

← bottom-up

To reduce γ_i to γ_{i-1} match some *rhs* β against γ_i then replace β with its corresponding *lhs*, A. *(assuming the production $A \rightarrow \beta$)*

In terms of the parse tree, this is working from leaves to root

- Nodes with no parent in a partial tree form its *upper fringe*
- Since each replacement of β with A shrinks the upper fringe, we call it a *reduction*.

The parse tree need not be built, it can be simulated

$$|\textit{parse tree nodes}| = |\textit{terminal symbols}| + |\textit{reductions}|$$

Finding Reductions

Consider the simple grammar

1	*Goal*	→	a *A* *B* e
2	*A*	→	*A* b c
3		|	b
4	*B*	→	d

And the input string abbcde

Sentential Form	*Next Reduction Prod'n*	*Next Reduction Pos'n*
abbcde	3	2
a *A* bcde	2	4
a *A* de	4	3
a *A* *B* e	1	4
Goal	—	—

The trick is scanning the input and finding the next reduction

The mechanism for doing this must be efficient

Finding Reductions (Handles)

The parser must find a substring β of the tree's frontier that *matches some production $A \rightarrow \beta$ that occurs as one step in the rightmost derivation* *($\Rightarrow \beta \rightarrow A$ is in RRD)*

Informally, we call this substring β a *handle*

Formally,

A *handle* of a right-sentential form γ is a pair $\langle A \rightarrow \beta, k \rangle$ where $A \rightarrow \beta \in P$ and k is the position in γ of β's rightmost symbol.

If $\langle A \rightarrow \beta, k \rangle$ is a handle, then replacing β at k with A produces the right sentential form from which γ is derived in the rightmost derivation.

Because γ is a right-sentential form, the substring to the right of a handle contains only terminal symbols

$\Rightarrow$ the parser doesn't need to scan past the handle *(very far)*

Finding Reductions (Handles)

Theorem:

If G is unambiguous, then every right-sentential form has a unique handle.

Sketch of Proof:

1. *G* is unambiguous ⇒ rightmost derivation is unique
2. ⇒ a unique production $A \rightarrow \beta$ applied to derive γ_i from γ_{i-1}
3. ⇒ a unique position ***k*** at which $A \rightarrow \beta$ is applied
4. ⇒ a unique handle $\langle A \rightarrow \beta, k \rangle$

This all follows from the definitions

If we can find those handles, we can build a derivation!

Example

(a very busy slide)

1	*Goal*	→	*Expr*
2	*Expr*	→	*Expr* + *Term*
3		|	*Expr* - *Term*
4		|	*Term*
5	*Term*	→	*Term* * *Factor*
6		|	*Term* / *Factor*
7		|	*Factor*
8	*Factor*	→	number
9		|	id
10		|	(*Expr*)

The expression grammar

Prod'n.	Sentential Form	Handle
—	*Goal*	—
1	*Expr*	1,1
3	*Expr* - *Term*	3,3
5	*Expr* - *Term* * *Factor*	5,5
9	*Expr* - *Term* * <id,y>	9,5
7	*Expr* - *Factor* * <id,y>	7,3
8	*Expr* - <num,2> * <id,y>	8,3
4	*Term* - <num,2> * <id,y>	4,1
7	*Factor* - <num,2> * <id,y>	7,1
9	<id,x> - <num,2> * <id,y>	9,1

Handles for rightmost derivation of x - 2 * y

This is the inverse of Figure 3.9 in EaC

Handle-pruning, Bottom-up Parsers

The process of discovering a handle & reducing it to the appropriate left-hand side is called *handle pruning*

Handle pruning forms the basis for a bottom-up parsing method

To construct a rightmost derivation

$$S \Rightarrow \gamma_0 \Rightarrow \gamma_1 \Rightarrow \gamma_2 \Rightarrow \ldots \Rightarrow \gamma_{n-1} \Rightarrow \gamma_n \Rightarrow w$$

Apply the following simple algorithm

for $i \leftarrow n$ *to 1 by –1*

Find the handle $\langle A_i \rightarrow \beta_i , \mathbf{k}_i \rangle$ in γ_i

Replace β_i *with* A_i *to generate* γ_{i-1}

This takes *2n* steps

of course, *n* is unknown until the derivation is built

Shift-reduce Parsing

Shift reduce parsers are easily built and easily understood

A shift-reduce parser is a stack automaton with four actions

- *Shift* — next word is shifted onto the stack
- *Reduce* — right end of handle is at top of stack
  - Locate left end of handle within the stack
  - Pop handle off stack & push appropriate *lhs*
- *Accept* — stop parsing & report success
- *Error* — call an error reporting/recovery routine

Accept & *Error* are simple

Shift is just a push and a call to the scanner

Reduce takes |*rhs*| pops & 1 push

But how do you know when to shift and when to reduce?

Handle-pruning, Bottom-up Parsers

A simple *shift-reduce parser*:

```
push INVALID
token ← next_token( )
repeat until (top of stack = Goal and token = EOF)
    if the top of the stack is a handle A→β
        then      // reduce β to A
            pop |β| symbols off the stack
            push A onto the stack
        else if (token ≠ EOF)
            then // shift
                push token
                token ← next_token( )
        else     // need to shift, but out of input
            report an error
```

How do errors show up?

- failure to find a handle
- hitting EOF & needing to shift (final else clause)

Either generates an error

Figure 3.7 in EAC

Back to x - 2 * y

Stack	Input	Handle	Action
$	id – num * id	*none*	shift
$ id	– num * id		

1. **Shift until the top of the stack is the right end of a handle**
2. **Find the left end of the handle & reduce**

Back to x - 2 * y

Stack	Input	Handle	Action
$	id – num * id	*none*	shift
$ id	– num * id	9,1	red. 9
$ *Factor*	– num * id	7,1	red. 7
$ *Term*	– num * id	4,1	red. 4
$ *Expr*	– num * id		

1. **Shift until the top of the stack is the right end of a handle**
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$ id	– num * id	9,1	red. 9
$ *Factor*	– num * id	7,1	red. 7
$ *Term*	– num * id	4,1	red. 4
$ *Expr*	– num * id	*none*	shift
$ *Expr* –	num * id	*none*	shift
$ *Expr* – num	* id		

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Back to x - 2 * y

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$ *Factor*	– num * id	7,1	red. 7
$ *Term*	– num * id	4,1	red. 4
$ *Expr*	– num * id	*none*	shift
$ *Expr* –	num * id	*none*	shift
$ *Expr* – num	* id	8,3	red. 8
$ *Expr* – *Factor*	* id	7,3	red. 7
$ *Expr* – *Term*	* id		

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$ *Expr* – *Factor*	* id	7,3	red. 7
$ *Expr* – *Term*	* id	*none*	shift
$ *Expr* – *Term* *	id	*none*	shift
$ *Expr* – *Term* * id			

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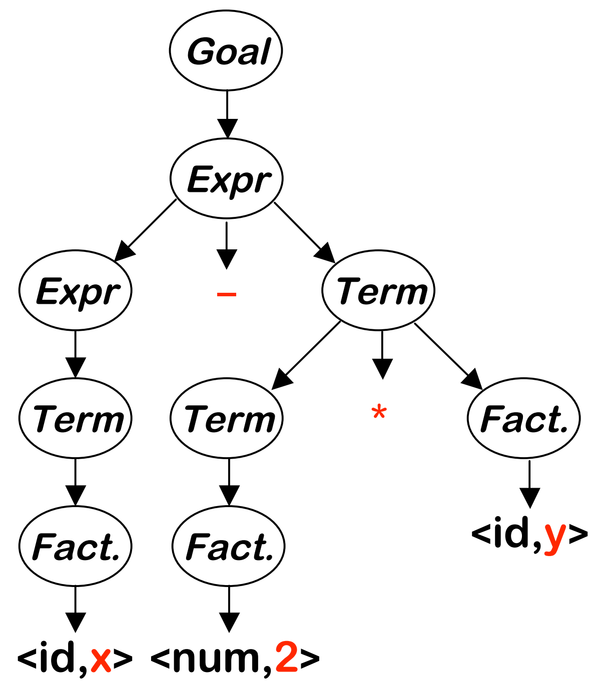
$ *Expr* – *Term*	* id	*none*	shift
$ *Expr* – *Term* *	id	*none*	shift
$ *Expr* – *Term* * id		9,5	red. 9
$ *Expr* – *Term* * *Factor*		5,5	red. 5
$ *Expr* – *Term*		3,3	red. 3
$ *Expr*		1,1	red. 1
$ *Goal*		*none*	accept

5 shifts + 9 reduces + 1 accept

1. **Shift until the top of the stack is the right end of a handle**
2. **Find the left end of the handle & reduce**

Example

Stack	Input	Action
$	id – num * id	shift
$ id	– num * id	red. 9
$ *Factor*	– num * id	red. 7
$ *Term*	– num * id	red. 4
$ *Expr*	– num * id	shift
$ *Expr* –	num * id	shift
$ *Expr* – num	* id	red. 8
$ *Expr* – *Factor*	* id	red. 7
$ *Expr* – *Term*	* id	shift
$ *Expr* – *Term* *	id	shift
$ *Expr* – *Term* * id		red. 9
$ *Expr* – *Term* * *Factor*		red. 5
$ *Expr* – *Term*		red. 3
$ *Expr*		red. 1
$ *Goal*		accept



An Important Lesson about Handles

To be a handle, a substring of a sentential form γ must have two properties:

- It must match the right hand side β of some rule $A \rightarrow \beta$
- There must be some rightmost derivation from the goal symbol that produces the sentential form γ with $A \rightarrow \beta$ as the last production applied

- Simply looking for right hand sides that match strings is not good enough
- Critical Question: How can we know when we have found a handle without generating lots of different derivations?
  - Answer: we use look ahead in the grammar along with tables produced as the result of analyzing the grammar.
  - *LR(1)* parsers build a DFA that runs over the stack & finds them

Ken's slides on operator precedence are online

Extra Slides Start Here

An Important Lesson about Handles

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- Simply looking for right hand sides that match strings is not good enough
- Critical Question: How can we know when we have found a handle without generating lots of different derivations?
  - Answer: we use lookahead in the grammar along with tables produced as the result of analyzing the grammar.
    - There are a number of different ways to do this.
    - We will look at two: *operator precedence* and *LR* parsing