Parsing III

(Top-down parsing: recursive descent & *LL(1)*)

Roadmap (Where are we?)

We set out to study parsing

- Specifying syntax
  - → Context-free grammars √
  - → Ambiguity √
- Top-down parsers
  - → Algorithm & its problem with left recursion √
  - → Left-recursion removal √
- Predictive top-down parsing
  - → The LL(1) condition today
  - → Simple recursive descent parsers today
  - → Table-driven LL(1) parsers today

Picking the "Right" Production

If it picks the wrong production, a top-down parser may backtrack

Alternative is to look ahead in input & use context to pick correctly

How much lookahead is needed?

- In general, an arbitrarily large amount
- Use the Cocke-Younger, Kasami algorithm or Earley's algorithm

Fortunately,

- Large subclasses of CFGs can be parsed with limited lookahead
- Most programming language constructs fall in those subclasses

Among the interesting subclasses are *LL(1)* and *LR(1)* grammars

Predictive Parsing

Basic idea

Given $A \rightarrow \alpha \mid \beta$, *the parser should be able to choose between* α *&* β

FIRST sets

For some *rhs* $\alpha \in G$, define FIRST(α) as the set of tokens that appear as the first symbol in some string that derives from α

That is, $\underline{x} \in$ FIRST(α) *iff* $\alpha \Rightarrow^* \underline{x}\,\gamma$, for some γ

We will defer the problem of how to compute FIRST sets until we look at the *LR(1)* table construction algorithm

Predictive Parsing

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The LL(1) Property

If $A \rightarrow \alpha$ and $A \rightarrow \beta$ both appear in the grammar, we would like

$$\text{FIRST}(\alpha) \cap \text{FIRST}(\beta) = \varnothing$$

This would allow the parser to make a correct choice with a lookahead of exactly one symbol !

This is almost correct
See the next slide

Predictive Parsing

What about ε-productions?

$\Rightarrow$ They complicate the definition of LL(1)

If $A \rightarrow \alpha$ and $A \rightarrow \beta$ and $\varepsilon \in$ FIRST(α), then we need to ensure that FIRST(β) is disjoint from FOLLOW(α), too

Define FIRST^{+}(α) as

- FIRST(α) $\cup$ FOLLOW(α), if $\varepsilon \in$ FIRST(α)
- FIRST(α), otherwise

Then, a grammar is *LL(1)* iff $A \rightarrow \alpha$ and $A \rightarrow \beta$ implies

FIRST^{+}(α) $\cap$ FIRST^{+}(β) = $\emptyset$

FOLLOW(α) is the set of all words in the grammar that can legally appear immediately after an α

Predictive Parsing

Given a grammar that has the *LL(1)* property

- Can write a simple routine to recognize each *lhs*
- Code is both simple & fast

Consider $A \rightarrow \beta_1 \mid \beta_2 \mid \beta_3$, with

$$\text{FIRST}^+(\beta_1) \cap \text{FIRST}^+(\beta_2) \cap \text{FIRST}^+(\beta_3) = \varnothing$$

```
/* find an A */
if (current_word ∈ FIRST(β1))
   find a β1 and return true
else if (current_word ∈ FIRST(β2))
   find a β2 and return true
else if (current_word ∈ FIRST(β3))
   find a β3 and return true
else
   report an error and return false
```

Of course, there is more detail to "find a β_i" (§ 3.3.4 in EAC)

Grammars with the *LL(1)* property are called *predictive grammars* because the parser can "predict" the correct expansion at each point in the parse.

Parsers that capitalize on the *LL(1)* property are called *predictive parsers*.

One kind of predictive parser is the *recursive descent* parser.

Recursive Descent Parsing

Recall the expression grammar, after transformation

1	*Goal*	$\rightarrow$	*Expr*
2	*Expr*	$\rightarrow$	*Term Expr′*
3	*Expr′*	$\rightarrow$	+ *Term Expr′*
4		|	– *Term Expr′*
5		|	ε
6	*Term*	$\rightarrow$	*Factor Term′*
7	*Term′*	$\rightarrow$	* *Factor Term′*
8		|	/ *Factor Term′*
9		|	ε
10	*Factor*	$\rightarrow$	<u>number</u>
11		|	<u>id</u>

This produces a parser with six <u>*mutually recursive*</u> routines:

- *Goal*
- *Expr*
- *EPrime*
- *Term*
- *TPrime*
- *Factor*

Each recognizes one *NT* or *T*

The term <u>*descent*</u> refers to the direction in which the parse tree is built.

Recursive Descent Parsing (Procedural)

A couple of routines from the expression parser

```
Goal( )
    token ← next_token( );
    if (Expr( ) = true & token = EOF)
       then next compilation step;
       else
           report syntax error;
           return false;

Expr( )
   if (Term( ) = false)
     then return false;
     else return Eprime( );
```

looking for EOF, found token

```
Factor( )
  if (token = Number) then
     token ← next_token( );
     return true;
  else if (token = Identifier) then
      token ← next_token( );
      return true;
  else
     report syntax error;
     return false;
```

***EPrime*, *Term*, & *TPrime* follow the same basic lines (Figure 3.7, EAC)**

looking for Number or Identifier, found token instead

Recursive Descent Parsing

To build a parse tree:

- Augment parsing routines to build nodes
- Pass nodes between routines using a stack
- Node for each symbol on *rhs*
- Action is to pop *rhs* nodes, make them children of *lhs* node, and push this subtree

To build an abstract syntax tree

- Build fewer nodes
- Put them together in a different order

```
Expr( )
    result ← true;
    if (Term( ) = false)
      then return false;
      else if (EPrime( ) = false)
              then result ← false;
              else
               build an Expr node
               pop EPrime node
                  pop Term node
               make EPrime &
Term
                   children of Expr
               push Expr node
    return result;
```

Success ⇒ build a piece of the parse tree

This is a preview of Chapter 4

Left Factoring

What if my grammar does not have the LL(1) property?

$\Rightarrow$ Sometimes, we can transform the grammar

The Algorithm

$\forall$ *A* $\in$ *NT*,

find the longest prefix α *that occurs in two or more right-hand sides of A*

if $\alpha \neq \varepsilon$ *then replace all of the A productions,*

$A \rightarrow \alpha\beta_1 \mid \alpha\beta_2 \mid \ldots \mid \alpha\beta_n \mid \gamma$,

with

$A \rightarrow \alpha\, Z \mid \gamma$

$Z \rightarrow \beta_1 \mid \beta_2 \mid \ldots \mid \beta_n$

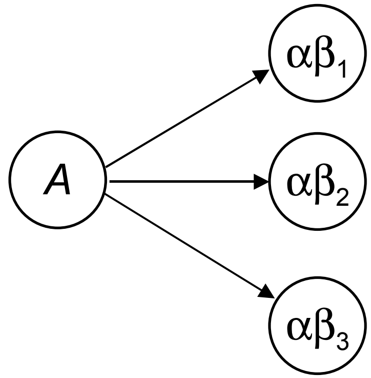
where Z is a new element of NT

Repeat until no common prefixes remain

Left Factoring

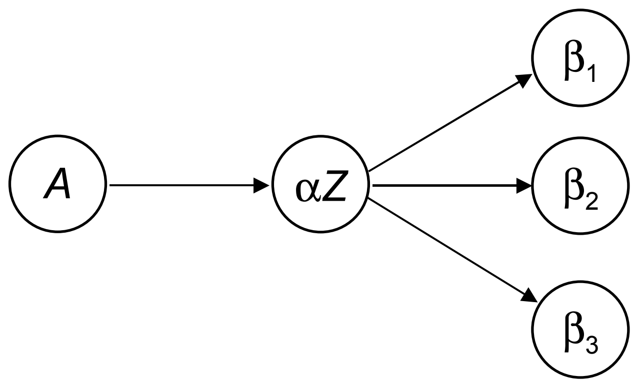
A graphical explanation for the same idea

$A \rightarrow \alpha\beta_1$
$\quad | \ \alpha\beta_2$
$\quad | \ \alpha\beta3$



becomes …

$A \rightarrow \alpha \ Z$
$Z \rightarrow \beta_1$
$\quad | \ \ \beta_2$
$\quad | \ \ \beta_n$



Left Factoring (An example)

Consider the following fragment of the expression grammar

Factor	$\rightarrow$	Identifier
	|	Identifier [*ExprList*]
	|	Identifier (*ExprList*)

FIRST(rhs_1) = { Identifier }
FIRST(rhs_2) = { Identifier }
FIRST(rhs_3) = { Identifier }

After left factoring, it becomes

Factor	$\rightarrow$	Identifier*Arguments*
Argumen	$\rightarrow$	[*ExprList*]
	|	(*ExprList*)
	|	ε

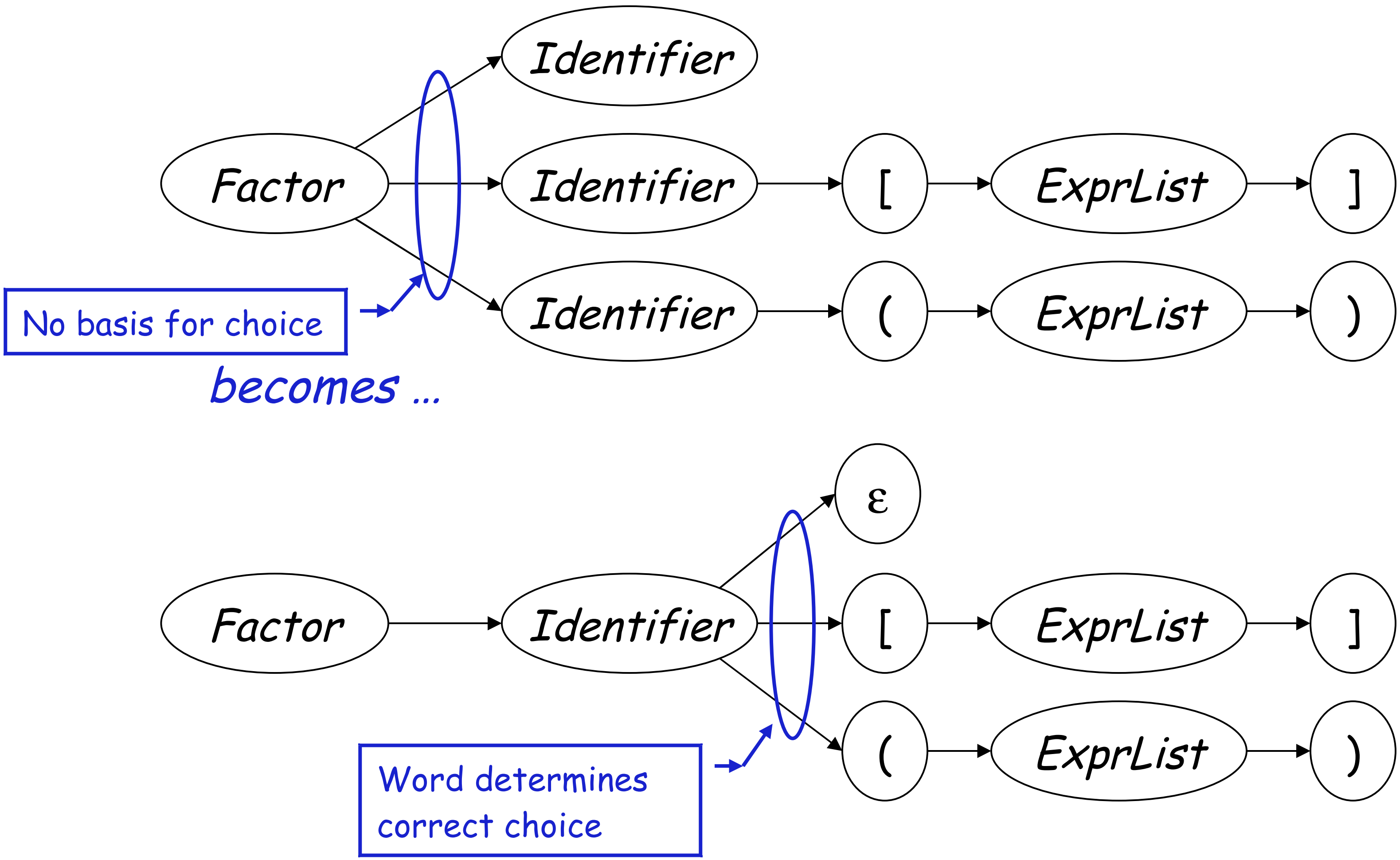
FIRST(rhs_1) = { Identifier }
FIRST(rhs_2) = { [}
FIRST(rhs_3) = { (}
FIRST(rhs_4) = FOLLOW(*Factor*)

$\Rightarrow$ It has the *LL(1)* property

This form has the same syntax, with the *LL(1)* property

Left Factoring

Graphically



Left Factoring (Generality)

Question

By *eliminating left recursion* and *left factoring*, can we transform an arbitrary CFG to a form where it meets the *LL(1)* condition? (and can be parsed predictively with a single token lookahead?)

Answer

Given a CFG that doesn't meet the *LL(1)* condition, it is undecidable whether or not an equivalent *LL(1)* grammar exists.

Example

$\{a^n 0\ b^n \mid n \geq 1\} \cup \{a^n 1\ b^{2n} \mid n \geq 1\}$ has no *LL(1)* grammar

Language that Cannot Be LL(1)

Example

$\{a^n 0\, b^n \mid n \geq 1\} \cup \{a^n 1\, b^{2n} \mid n \geq 1\}$ has no *LL(1)* grammar

$$
\begin{aligned}
G &\rightarrow \underline{a}A\underline{b} \\
&\mid \underline{a}B\underline{bb} \\
A &\rightarrow \underline{a}A\underline{b} \\
&\mid \underline{0} \\
B &\rightarrow \underline{a}B\underline{bb} \\
&\mid \underline{1}
\end{aligned}
$$

Problem: need an unbounded number of a characters before you can determine whether you are in the A group or the B group.

Recursive Descent (Summary)

1. Build FIRST (and FOLLOW) sets
2. Massage grammar to have *LL(1)* condition
   a. Remove left recursion
   b. Left factor it
3. Define a procedure for each non-terminal
   a. Implement a case for each right-hand side
   b. Call procedures as needed for non-terminals
4. Add extra code, as needed
   a. Perform context-sensitive checking
   b. Build an IR to record the code

Can we automate this process?

FIRST and FOLLOW Sets

FIRST(α)

For some $\alpha \in T \cup NT$, define FIRST(α) as the set of tokens that appear as the first symbol in some string that derives from α

That is, $\underline{x} \in$ FIRST(α) *iff* $\alpha \Rightarrow^* \underline{x}\,\gamma$, for some γ

FOLLOW(α)

For some $\alpha \in NT$, define FOLLOW(α) as the set of symbols that can occur immediately after α in a valid sentence.

FOLLOW(S) = {EOF}, where S is the start symbol

To build FIRST sets, we need FOLLOW sets …

Computing FOLLOW Sets

FOLLOW(S) $\leftarrow$ *{EOF }*

for each A $\in$ *NT, FOLLOW(A)* $\leftarrow$ *Ø*

while (FOLLOW sets are still changing)

    for each p $\in$ *P, of the form* $A \rightarrow \beta_1\beta_2 \ldots \beta_k$

        $FOLLOW(\beta_k) \leftarrow FOLLOW(\beta_k) \cup FOLLOW(A)$

        $TRAILER \leftarrow FOLLOW(A)$

        for i $\leftarrow$ *k down to 2*

            if $\varepsilon \in FIRST(\beta_i)$ *then*

                $FOLLOW(\beta_{i-1}) \leftarrow FOLLOW(\beta_{i-1}) \cup \{ FIRST(\beta_i) - \{ \varepsilon \} \}$

                $\cup\ TRAILER$

            else

                $FOLLOW(\beta_{i-1}) \leftarrow FOLLOW(\beta_{i-1}) \cup FIRST(\beta_i)$

                $TRAILER \leftarrow Ø$

Computing FIRST Sets

for each $x \in T$, *FIRST*(x) $\leftarrow$ { x }
for each $A \in NT$, *FIRST*(A) $\leftarrow$ Ø

while (*FIRST sets are still changing*)
    for each $p \in P$, *of the form* $A \rightarrow \beta$,
        if β *is* ε *then*
            FIRST(A) $\leftarrow$ *FIRST*(A) $\cup$ { ε }
        else if β *is* $B_1B_2 \ldots B_k$ *then begin*
            FIRST(A) $\leftarrow$ *FIRST*(A) $\cup$ (*FIRST*(B_1) – { ε })
            for $i \leftarrow 1$ *to* k–1 *by* 1 *while* $\varepsilon \in$ *FIRST*(B_i)
                FIRST(A) $\leftarrow$ *FIRST*(A) $\cup$ (*FIRST*(B_{i+1}) – { ε })
            if $i = k$–1 *and* $\varepsilon \in$ *FIRST*(B_k)
                then FIRST(A) $\leftarrow$ *FIRST*(A) $\cup$ { ε }
             end
for each $A \in NT$
    if $\varepsilon \in$ *FIRST*(A) *then*
        FIRST(A) $\leftarrow$ *FIRST*(A) $\cup$ *FOLLOW*(A)

Building Top-down Parsers

Given an *LL(1)* grammar, and its FIRST & FOLLOW sets …

- Emit a routine for each non-terminal
  - → Nest of if-then-else statements to check alternate rhs's
  - → Each returns true on success and throws an error on false
  - → Simple, working *(, perhaps ugly,)* code
- This automatically constructs a recursive-descent parser

I don't know of a system that does this …

Improving matters

- Nest of if-then-else statements may be slow
  - → Good case statement implementation would be better
- What about a table to encode the options?
  - → Interpret the table with a skeleton, as we did in scanning

Building Top-down Parsers

Strategy

- Encode knowledge in a table
- Use a standard "skeleton" parser to interpret the table

Example

- The non-terminal *Factor* has three expansions
  - → (*Expr*) or Identifier or Number
- Table might look like:

Terminal Symbols

Non-terminal Symbols

	+	-	*	/	Id.	Num.	EOF
Factor	—	—	—	—	10	11	—

Error on `+'

Reduce by rule 10 on `+'

Building Top Down Parsers

Building the complete table

- Need a row for every *NT* & a column for every *T*
- Need a table-driven interpreter for the table

LL(1) Skeleton Parser

```
token ← next_token()
push EOF onto Stack
push the start symbol, S, onto Stack
TOS ← top of Stack
loop forever
  if TOS = EOF and token = EOF then
     break & report success
  else if TOS is a terminal then
     if TOS matches token then
        pop Stack             // recognized TOS
        token ← next_token()
     else report error looking for TOS
  else                        // TOS is a non-terminal
     if TABLE[TOS,token] is A→ B1B2…Bk then
        pop Stack             // get rid of A
        push Bk, Bk-1, …, B1  // in that order
     else report error expanding TOS
  TOS ← top of Stack
```

exit on success

Building Top Down Parsers

Building the complete table

- Need a row for every *NT* & a column for every *T*
- Need an algorithm to build the table

Filling in TABLE[X,y], X $\in$ *NT*, y $\in$ *T*

1. entry is the rule $X \rightarrow \beta$, if y $\in$ FIRST(β)
2. entry is the rule $X \rightarrow \varepsilon$ if y $\in$ FOLLOW(X) and $X \rightarrow \varepsilon \in G$
3. entry is error if neither 1 nor 2 define it

If any entry is defined multiple times, *G* is not *LL(1)*

This is the *LL(1)* table construction algorithm

Extra Slides Start Here

Recursive Descent in Object-Oriented Languages

- Shortcomings of Recursive Descent
  - → Too procedural
  - → No convenient way to build parse tree
- Solution
  - → Associate a class with each non-terminal symbol
    - Allocated object contains pointer to the parse tree

```
Class NonTerminal {

public:
    NonTerminal(Scanner & scnr) { s = &scnr; tree = NULL; }
    virtual ~NonTerminal() { }
    virtual bool isPresent() = 0;
    TreeNode * abSynTree() { return tree; }

protected:
    Scanner * s;
    TreeNode * tree;
}
```

Non-terminal Classes

```
Class Expr : public NonTerminal {
public:
    Expr(Scanner & scnr) : NonTerminal(scnr) { }
    virtual bool isPresent();
}

Class EPrime : public NonTerminal {
public:
    EPrime(Scanner & scnr, TreeNode * p) :
        NonTerminal(scnr) { exprSofar = p; }
    virtual bool isPresent();
protected:
    TreeNode * exprSofar;
}

…  // definitions for Term and TPrime

Class Factor : public NonTerminal {
public:
    Factor(Scanner & scnr) : NonTerminal(scnr) { };
    virtual bool isPresent();
}
```

Implementation of isPresent

```
bool Expr::isPresent() {

    Term * operand1 = new Term(*s);
    if (!operand1->isPresent()) return FALSE;

    Eprime * operand2 = new EPrime(*s, NULL);
    if (!operand2->isPresent()) // do nothing;

    return TRUE;
}
```

Implementation of isPresent

```
bool EPrime::isPresent() {

    token_type op = s->nextToken();
    if (op == PLUS || op == MINUS) {
        s->advance();

        Term * operand2 = new Term(*s);
        if (!operand2->isPresent()) throw SyntaxError(*s);

        Eprime * operand3 = new EPrime(*s, NULL);
        if (operand3->isPresent()); //do nothing

        return TRUE;
    }
    else return FALSE;
}
```

Abstract Syntax Tree Construction

```
bool Expr::isPresent() { // with semantic processing

    Term * operand1 = new Term(*s);
    if (!operand1->isPresent()) return FALSE;
    tree = operand1->abSynTree();

    EPrime * operand2 = new EPrime(*s, tree);
    if (operand2->isPresent())
        tree  = operand2->absSynTree();

    // here tree is either the tree for the Term
    //      or the tree for Term followed by EPrime
    return TRUE;
}
```

Abstract Syntax Tree Construction

```
bool EPrime::isPresent() { // with semantic processing
    token_type op = s->nextToken();
    if (op == PLUS || op == MINUS) {
        s->advance();

        Term * operand2 = new Term(*s);
        if (!operand2->isPresent()) throw SyntaxError(*s);

        TreeNode * t2 = operand2->absSynTree();
        tree = new TreeNode(op, exprSofar, t2);

        Eprime * operand3 = new Eprime(*s, tree);
        if (operand3->isPresent())
            tree = operand3->absSynTree();
        return TRUE;
    }
    else return FALSE;
}
```

Factor

```
bool Factor::isPresent() { // with semantic processing
    token_type op = s->nextToken();

    if (op == IDENTIFIER | op == NUMBER) {
        tree = new TreeNode(op, s->tokenValue());
        s->advance();
        return TRUE;
    }
    else if (op == LPAREN) {
        s->advance();
        Expr * operand = new Expr(*s);
        if (!operand->isPresent()) throw SyntaxError(*s);
        if (s->nextToken() != RPAREN) throw SyntaxError(*s);
        s->advance();
        tree = operand->absSynTree();
        return TRUE;
    }
    else return FALSE;
}
```