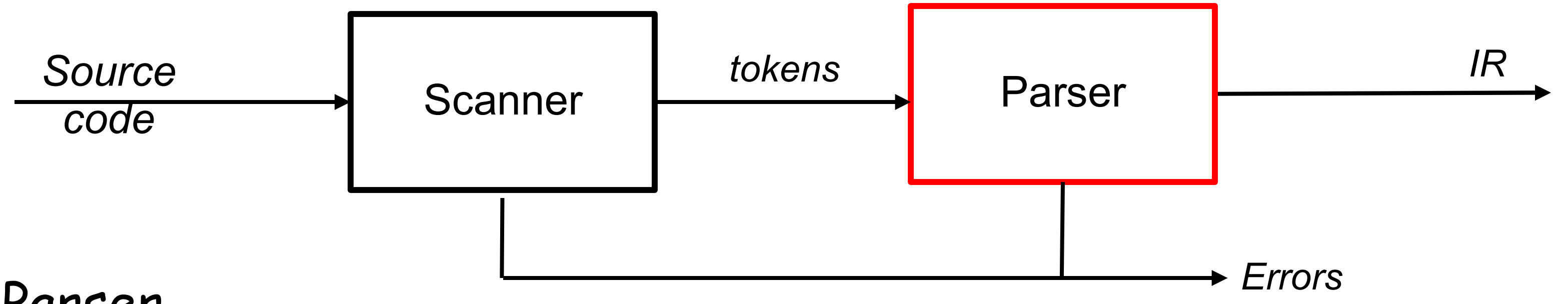
Introduction to Parsing

The Front End



Parser

- Checks the stream of words and their parts of speech (produced by the scanner) for grammatical correctness
- Determines if the input is syntactically well formed
- Guides checking at deeper levels than syntax
- Builds an IR representation of the code

Think of this as the mathematics of diagramming sentences

The Study of Parsing

The process of discovering a *derivation* for some sentence

- Need a mathematical model of syntax — a grammar *G*
- Need an algorithm for testing membership in *L(G)*
- Need to keep in mind that our goal is building parsers, not studying the mathematics of arbitrary languages

Roadmap

1 Context-free grammars and derivations

2 Top-down parsing

  → Hand-coded recursive descent parsers

3 Bottom-up parsing

  → Generated LR(1) parsers

Specifying Syntax with a Grammar

Context-free syntax is specified with a context-free grammar

SheepNoise → *SheepNoise* baa
| baa

This *CFG* defines the set of noises sheep normally make

It is written in a variant of Backus–Naur form

Formally, a grammar is a four tuple, $G = (S,N,T,P)$

- S is the *start symbol* *(set of strings in L(G))*
- N is a set of *non-terminal symbols* *(syntactic variables)*
- T is a set of *terminal symbols* *(words)*
- P is a set of *productions* or *rewrite rules* $(P : N \rightarrow (N \cup T)^{+})$

Example due to Dr. Scott K. Warren

Deriving Syntax

We can use the *SheepNoise* grammar to create sentences

- use the productions as *rewriting rules*

Rule	*Sentential Form*
Ñ	*SheepNoise*
2	baa

Rule	*Sentential Form*
Ñ	*SheepNoise*
1	*SheepNoise* baa
2	baa baa

Rule	*Sentential Form*
Ñ	*SheepNoise*
1	*SheepNoise* baa
1	*SheepNoise* baa baa
2	baa baa baa

And so on ...

While it is cute, this example quickly runs out of intellectual steam ...

A More Useful Grammar

To explore the uses of CFGs,we need a more complex grammar

1	*Expr*	→	*Expr Op Expr*
2		|	number
3		|	id
4	*Op*	→	+
5		|	–
6		|	*
7		|	/

Rule	*Sentential Form*
—	*Expr*
1	*Expr Op Expr*
2	<id,x> *Op Expr*
5	<id,x> – *Expr*
1	<id,x> – *Expr Op Expr*
2	<id,x> – <num,2> *Op Expr*
6	<id,x> – <num,2> * *Expr*
3	<id,x> – <num,2> * <id,y>

- Such a sequence of rewrites is called a *derivation*
- Process of discovering a derivation is called *parsing*

We denote this derivation: *Expr* $\Rightarrow^*$ id – num * id

Derivations

- At each step, we choose a non-terminal to replace
- Different choices can lead to different derivations

Two derivations are of interest

- *Leftmost derivation* — replace leftmost NT at each step
- *Rightmost derivation* — replace rightmost NT at each step

These are the two *systematic* derivations

(We don't care about randomly-ordered derivations!)

The example on the preceding slide was a *leftmost* derivation

- Of course, there is also a *rightmost* derivation
- Interestingly, it turns out to be different

The Two Derivations for x – 2 * y

Rule	Sentential Form
—	*Expr*
1	*Expr Op Expr*
3	<id,x> *Op Expr*
5	<id,x> – *Expr*
1	<id,x> – *Expr Op Expr*
2	<id,x> – <num,2> *Op Expr*
6	<id,x> – <num,2> * *Expr*
3	<id,x> – <num,2> * <id,y>

Leftmost derivation

Rule	Sentential Form
—	*Expr*
1	*Expr Op Expr*
3	*Expr Op* <id,y>
6	*Expr* * <id,y>
1	*Expr Op Expr* * <id,y>
2	*Expr Op* <num,2> * <id,y>
5	*Expr* – <num,2> * <id,y>
3	<id,x> – <num,2> * <id,y>

Rightmost derivation

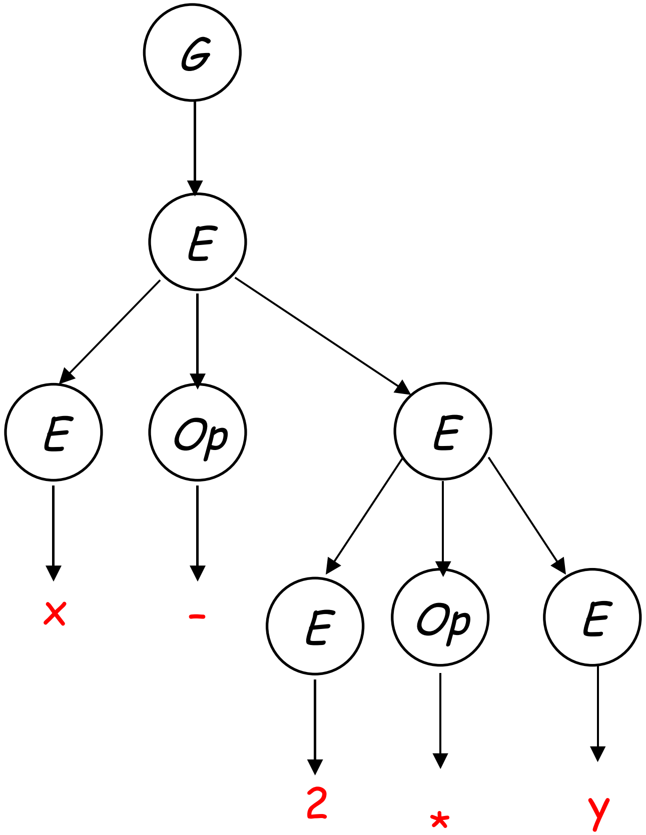
In both cases, *Expr* $\Rightarrow^*$ id – num * id

- The two derivations produce different parse trees
- The parse trees imply different evaluation orders!

Derivations and Parse Trees

Leftmost derivation

Rule	*Sentential Form*
—	*Expr*
1	*Expr Op Expr*
3	<id,x> *Op Expr*
5	<id,x> - *Expr*
1	<id,x> - *Expr Op Expr*
2	<id,x> - <num,2> *Op Expr*
6	<id,x> - <num,2> * *Expr*
3	<id,x> - <num,2> * <id,y>

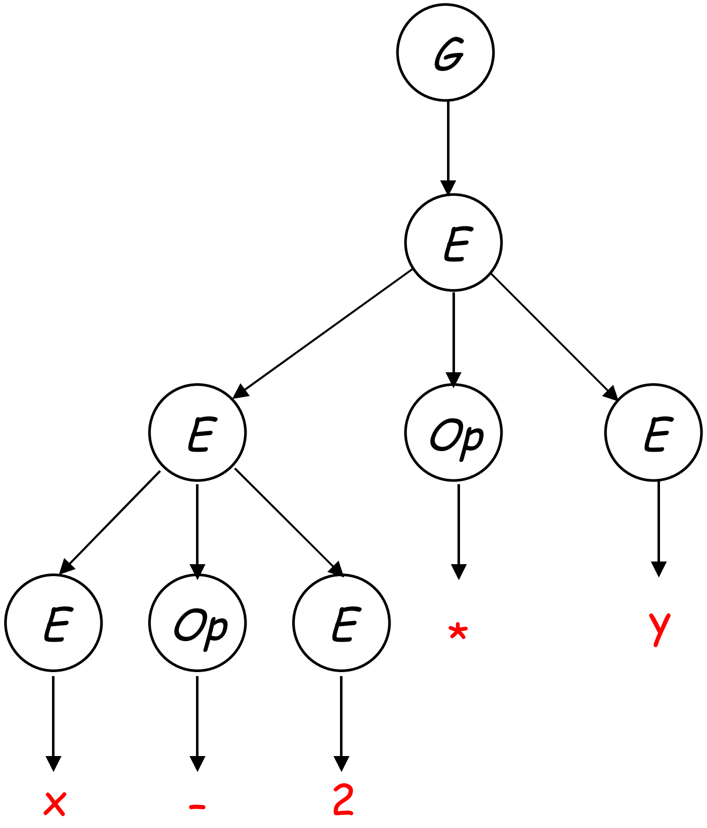


This evaluates as x - (2 * y)

Derivations and Parse Trees

Rightmost derivation

Rule	*Sentential Form*
—	*Expr*
1	*Expr Op Expr*
3	*Expr Op* <id,y>
6	*Expr* * <id,y>
1	*Expr Op Expr* * <id,y>
2	*Expr Op* <num,2> * <id,y>
5	*Expr* – <num,2> * <id,y>
3	<id,x> – <num,2> * <id,y>



This evaluates as (x - 2) * y

Derivations and Precedence

These two derivations point out a problem with the grammar:

It has no notion of precedence, or implied order of evaluation

To add precedence

- Create a non-terminal for each *level of precedence*
- Isolate the corresponding part of the grammar
- Force the parser to recognize high precedence subexpressions first

For algebraic expressions

- Multiplication and division, first (*level one*)
- Subtraction and addition, next (*level two*)

Derivations and Precedence

Adding the standard algebraic precedence produces:

	1	*Goal*	→	*Expr*
level two	2	*Expr*	→	*Expr* + *Term*
	3		|	*Expr* - *Term*
	4		|	*Term*
level one	5	*Term*	→	*Term* * *Factor*
	6		|	*Term* / *Factor*
	7		|	*Factor*
	8	*Factor*	→	number
	9		|	id

This grammar is slightly larger

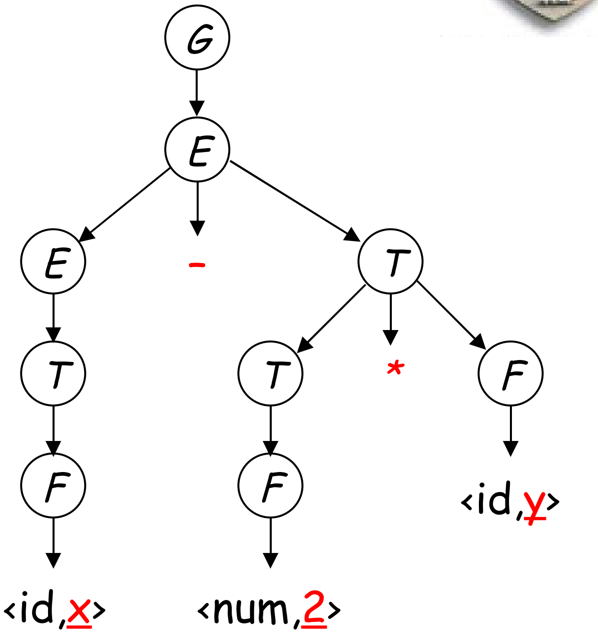
- Takes more rewriting to reach some of the terminal symbols
- Encodes expected precedence
- Produces same parse tree under leftmost & rightmost derivations

*Let's see how it parses x - 2 * y*

Derivations and Precedence

Rule	*Sentential Form*
—	*Goal*
1	*Expr*
3	*Expr* - *Term*
5	*Expr* - *Term* * *Factor*
9	Expr - Term * <id,y>
7	*Expr* - *Factor* * <id,y>
8	Expr- <num,2> * <id,y>
4	Term- <num,2> * <id,y>
7	*Factor* - <num,2> * <id,y>
9	<id,x> - <num,2> * <id,y>

The rightmost derivation



Its parse tree

This produces x – (2 * y), along with an appropriate parse tree.

Both the leftmost and rightmost derivations give the same expression, because the grammar directly encodes the desired precedence.

Ambiguous Grammars

Our original expression grammar had other problems

1	*Expr*	→	*Expr Op Expr*
2		|	number
3		|	id
4	*Op*	→	+
5		|	–
6		|	*
7		|	/

Rule	*Sentential Form*
—	*Expr*
1	*Expr Op Expr*
1	*Expr Op Expr Op Expr*
3	<id,x> *Op Expr Op Expr*
5	<id,x> – *Expr Op Expr*
2	<id,x> – <num,2> *Op Expr*
6	<id,x> – <num,2> * *Expr*
3	<id,x> – <num,2> * <id,y>

- This grammar allows multiple leftmost derivations for x - 2 * y
- Hard to automate derivation if > 1 choice
- The grammar is *ambiguous*

different choice than the first time

Two Leftmost Derivations for x - 2 * y

The Difference:

- Different productions chosen on the second step

Rule	Sentential Form
—	Expr
1	Expr Op Expr
③	<id,x> Op Expr
5	<id,x> - Expr
1	<id,x> - Expr Op Expr
2	<id,x> - <num,2> Op Expr
6	<id,x> - <num,2> * Expr
3	<id,x> - <num,2> * <id,y>

Original choice

Rule	Sentential Form
—	Expr
1	Expr Op Expr
①	Expr Op Expr Op Expr
3	<id,x> Op Expr Op Expr
5	<id,x> - Expr Op Expr
2	<id,x> - <num,2> Op Expr
6	<id,x> - <num,2> * Expr
3	<id,x> - <num,2> * <id,y>

New choice

- Both derivations succeed in producing x - 2 * y

Ambiguous Grammars

Definitions

- If a grammar has more than one leftmost derivation for a single *sentential form*, the grammar is *ambiguous*
- If a grammar has more than one rightmost derivation for a single sentential form, the grammar is *ambiguous*
- The leftmost and rightmost derivations for a sentential form may differ, even in an unambiguous grammar

Classic example — the *if*-*then*-*else* problem

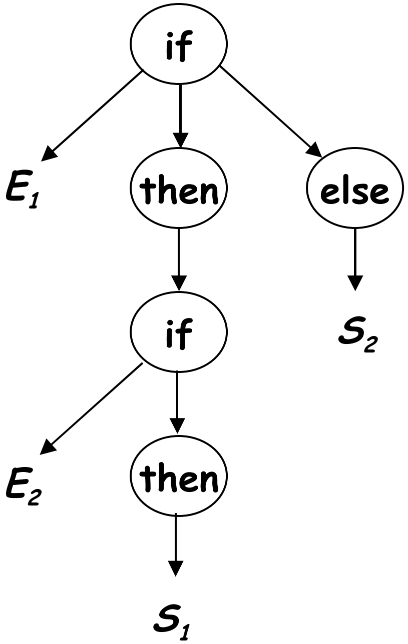
Stmt → if *Expr* then *Stmt*
| if *Expr* then *Stmt* else *Stmt*
| *… other stmts …*

This ambiguity is entirely grammatical in nature

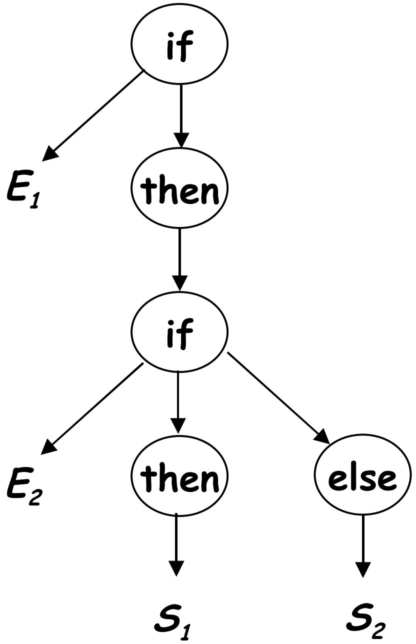
Ambiguity

This sentential form has two derivations

if $Expr_1$ then if $Expr_2$ then $Stmt_1$ else $Stmt_2$



production 2, then production 1



production 1, then production 2

Ambiguity

Removing the ambiguity

- Must rewrite the grammar to avoid generating the problem
- Match each else to innermost unmatched if *(common sense rule)*

1 *Stmt* → *WithElse*
2 | *NoElse*
3 *WithElse* → if *Expr* then *WithElse* else *WithElse*
4 | *OtherStmt*
5 *NoElse* → if *Expr* then *Stmt*
6 | if *Expr* then *WithElse* else *NoElse*

Intuition: a *NoElse* always has no else on its last cascaded *else if* statement

With this grammar, the example has only one derivation

Ambiguity

if $Expr_1$ then if $Expr_2$ then $Stmt_1$ else $Stmt_2$

Rule	*Sentential Form*
Ñ	*Stmt*
2	*NoElse*
5	if *Expr* then *Stmt*
?	if E_1 then *Stmt*
1	if E_1 then *WithElse*
3	if E_1 then if *Expr* then *WithElse* else *WithElse*
?	if E_1 then if E_2 then *WithElse* else *WithElse*
4	if E_1 then if E_2 then S_1 else *WithElse*
4	if E_1 then if E_2 then S_1 else S_2

This binds the else controlling S_2 to the inner if

Deeper Ambiguity

Ambiguity usually refers to confusion in the CFG

Overloading can create deeper ambiguity

`a = f(17)`

In many Algol-like languages, f could be either a function or a subscripted variable

Disambiguating this one requires context

- Need values of declarations
- Really an issue of *type*, not context-free syntax
- Requires an extra-grammatical solution (not in CFG)
- Must handle these with a different mechanism
  - → Step outside grammar rather than use a more complex grammar

Ambiguity - the Final Word

Ambiguity arises from two distinct sources

- Confusion in the context-free syntax (*if*-*then*-*else*)
- Confusion that requires context to resolve (*overloading*)

Resolving ambiguity

- To remove context-free ambiguity, rewrite the grammar
- To handle context-sensitive ambiguity takes cooperation
  - → Knowledge of declarations, types, …
  - → Accept a superset of *L(G)* & check it by other means[†]
  - → This is a language design problem

Sometimes, the compiler writer accepts an ambiguous grammar

- → Parsing techniques that "do the right thing"
- → *i.e.*, always select the same derivation

[†] See Chapter 4

Parsing Techniques

Top-down parsers *(LL(1), recursive descent)*

- Start at the root of the parse tree and grow toward leaves
- Pick a production & try to match the input
- Bad "pick" ⇒ may need to backtrack
- Some grammars are backtrack-free *(predictive parsing)*

Bottom-up parsers *(LR(1), operator precedence)*

- Start at the leaves and grow toward root
- As input is consumed, encode possibilities in an internal state
- Start in a state valid for legal first tokens
- Bottom-up parsers handle a large class of grammars

Top-down Parsing

A top-down parser starts with the root of the parse tree

The root node is labeled with the goal symbol of the grammar

Top-down parsing algorithm:

Construct the root node of the parse tree

Repeat until the fringe of the parse tree matches the input string

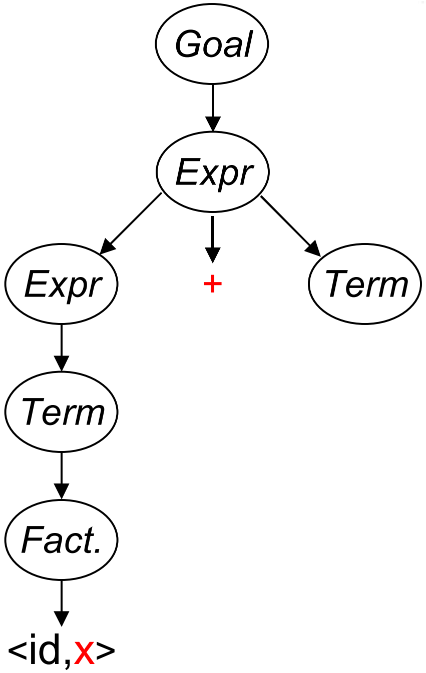
1. *At a node labeled A, select a production with A on its lhs and, for each symbol on its rhs, construct the appropriate child*
2. *When a terminal symbol is added to the fringe and it doesn't match the fringe, backtrack*
3. *Find the next node to be expanded* (label $\in$ NT)

- The key is picking the right production in step 1
  - → *That choice should be guided by the input string*

Example

Let's try x - 2 * y :

Rule	*Sentential Form*	*Input*
—	*Goal*	↑x - 2 * y
1	*Expr*	↑x - 2 * y
2	*Expr* + *Term*	↑x - 2 * y
4	*Term* + *Term*	↑x - 2 * y
7	*Factor* + *Term*	↑x - 2 * y
9	<id,x> + *Term*	↑x - 2 * y
9	<id,x> + *Term*	x ↑- 2 * y

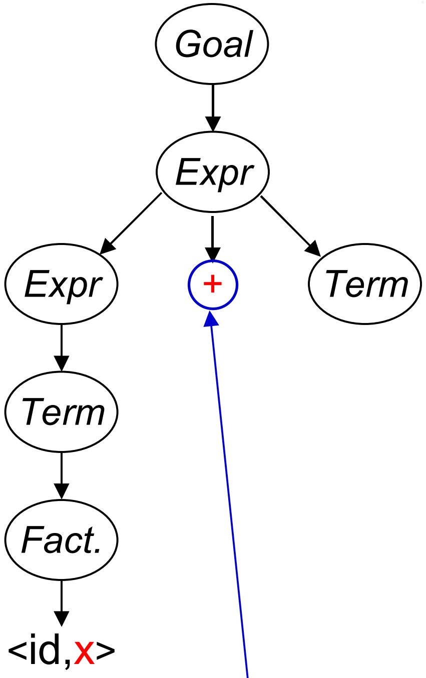


Leftmost derivation, choose productions in an order that exposes problems

Example

Let's try x – 2 * y :

Rule	Sentential Form	Input
—	*Goal*	↑x - 2 * y
1	*Expr*	↑x - 2 * y
2	*Expr* + *Term*	↑x - 2 * y
4	*Term* + *Term*	↑x - 2 * y
7	*Factor* + *Term*	↑x - 2 * y
9	<id,x> + *Term*	↑x - 2 * y
9	<id,x> + *Term*	x ↑- 2 * y



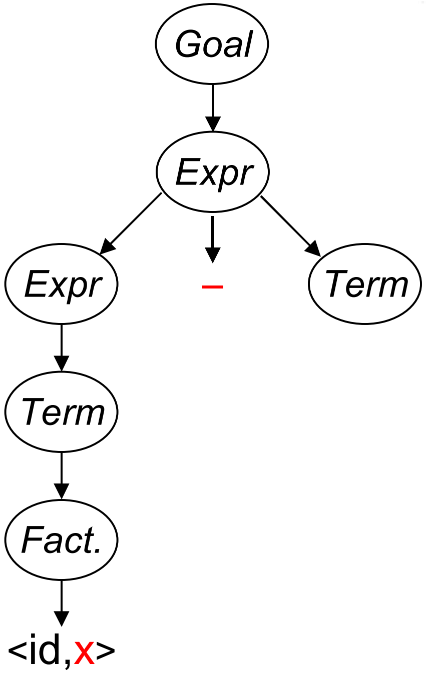
This worked well, except that "–" doesn't match "+"

The parser must backtrack to here

Example

Continuing with x - 2 * y :

Rule	Sentential Form	Input
—	*Goal*	↑x – 2 * y
1	*Expr*	↑x – 2 * y
3	*Expr – Term*	↑x – 2 * y
4	*Term – Term*	↑x – 2 * y
7	*Factor – Term*	↑x - 2 * y
9	<id,x> - *Term*	↑x – 2 * y
9	<id,x> – *Term*	x ↑– 2 * y
—	<id,x> - *Term*	x – ↑2 * y

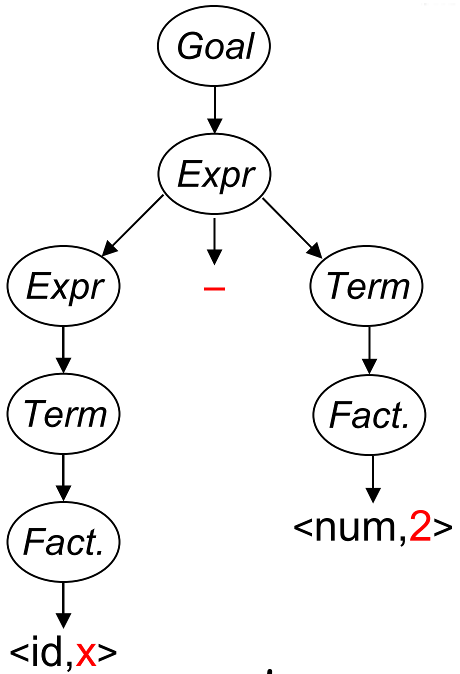


This time, “–” and “–” matched

We can advance past “–” to look at “2”

Example

Trying to match the “2” in x - 2 * y :



Rule	*Sentential Form*	*Input*
—	<id,x> - *Term*	x - ↑2 * y
7	<id,x> - *Factor*	x - ↑2 * y
9	<id,x> - <num,2>	x - ↑2 * y
—	<id,x> - <num,2>	x - 2 ↑* y

Where are we?

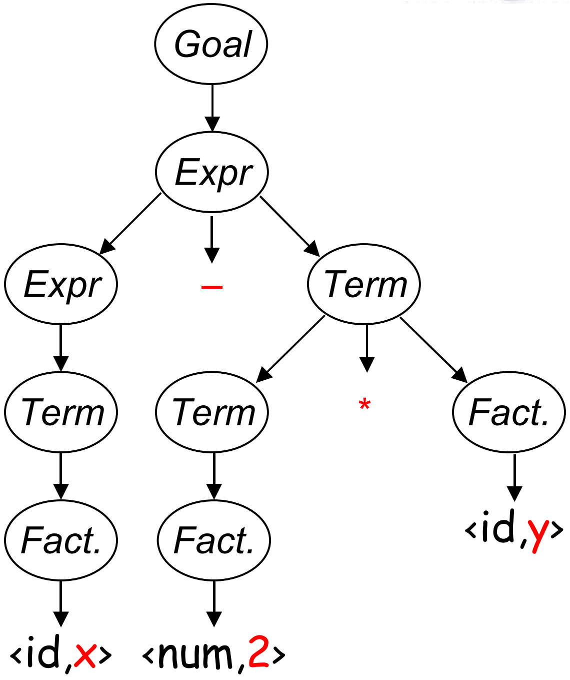
- “2” matches “2”
- We have more input, but no *NTs* left to expand
- The expansion terminated too soon

⇒ Need to backtrack

Example

Trying again with "2" in x - 2 * y :

Rule	*Sentential Form*	*Input*
—	<id,x> - *Term*	x - ↑2 * y
5	<id,x> - *Term* * *Fac*	x - ↑2 * y
7	<id,x> - *Factor* * *Fa*	x - ↑2 * y
8	<id,x> - <num,2> * *Fa*	x - ↑2 * y
—	<id,x> - <num,2> * *Fa*	x - 2 ↑* y
—	<id,x> - <num,2> * *Fa*	x - 2 * ↑y
9	<id,x> - <num,2> * <id	x - 2 * ↑y
—	<id,x> - <num,2> * <id	x - 2 * y↑



This time, we matched & consumed all the input

⇒ Success!

Left Recursion

Top-down parsers cannot handle left-recursive grammars

Formally,

A grammar is *left recursive* if $\exists\ A \in NT$ such that

$\exists$ a derivation $A \Rightarrow^{+} A\alpha$, for some string $\alpha \in (NT \cup T)^{+}$

Our expression grammar is left recursive

- This can lead to non-termination in a top-down parser
- For a top-down parser, any recursion must be right recursion
- We would like to convert the left recursion to right recursion

Non-termination is a bad property in any part of a compiler

Eliminating Left Recursion

To remove left recursion, we can transform the grammar

Consider a grammar fragment of the form

$$\begin{aligned} \mathit{Fee} &\rightarrow \mathit{Fee}\ \alpha \\ &\mid\ \beta \end{aligned}$$

where neither α nor β start with *Fee*

We can rewrite this as

$$\begin{aligned} \mathit{Fee} &\rightarrow \beta\ \mathit{Fie} \\ \mathit{Fie} &\rightarrow \alpha\ \mathit{Fie} \\ &\mid\ \varepsilon \end{aligned}$$

where *Fie* is a new non-terminal

This accepts the same language, but uses only right recursion