Instruction Selection, II
Tree-pattern matching

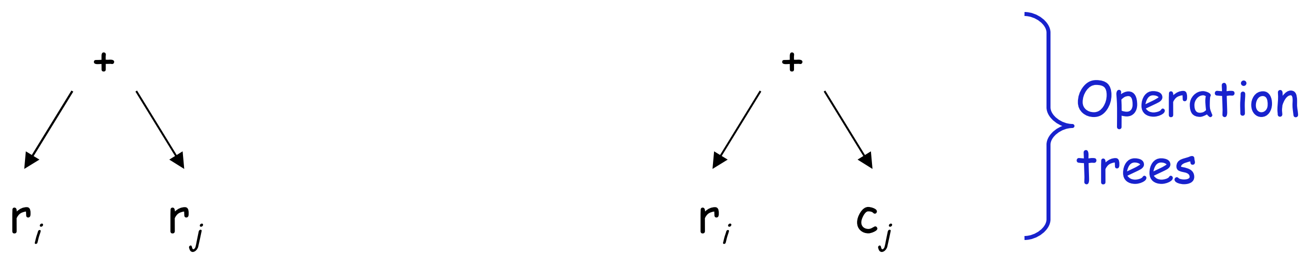
The Concept

Many compilers use tree-structured IRs

- Abstract syntax trees generated in the parser
- Trees or DAGs for expressions

These systems might well use trees to represent target ISA

Consider the ILOC add operators

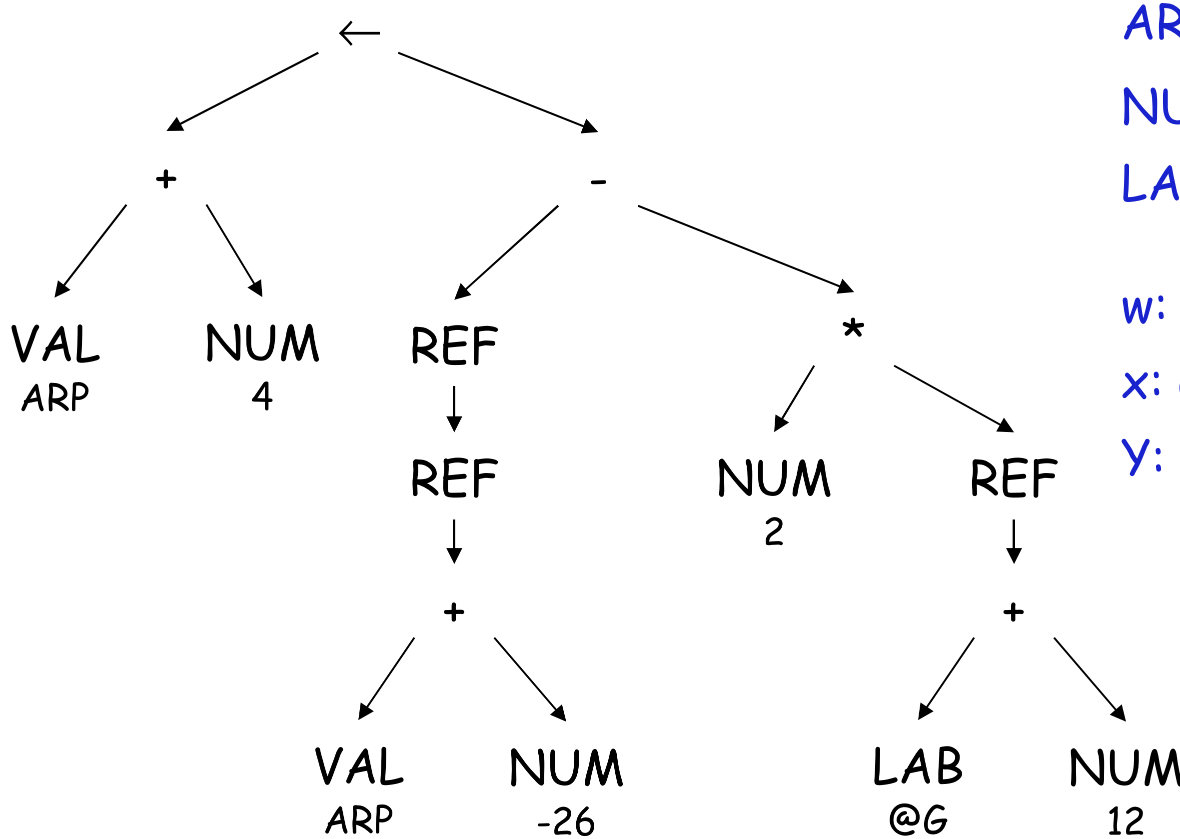


add $r_i, r_j \Rightarrow r_k$ addI $r_i, c_j \Rightarrow r_k$

If we can match these “pattern trees” against IR trees, …

The Concept

Low-level AST for $w \leftarrow x - 2 * y$



ARP: r_{arp}

NUM: constant

LAB: ASM label

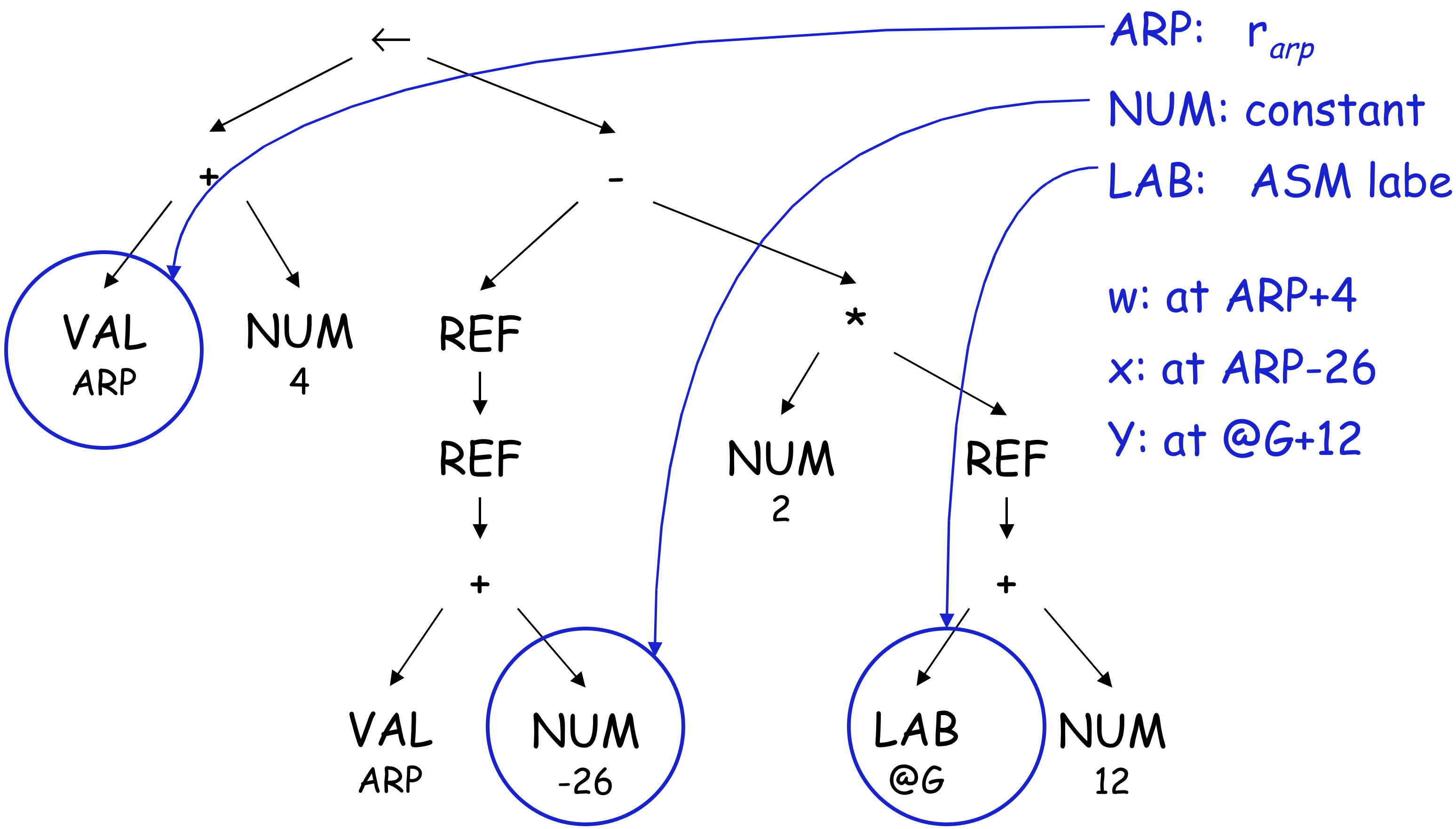
w: at ARP+4

x: at ARP-26

y: at @G+12

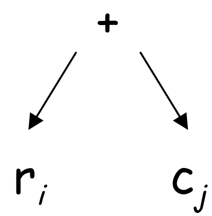
The Concept

Low-level AST for w ← x - 2 * y

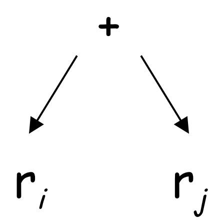


Notation

To describe these trees, we need a concise notation



$+(r_i, c_j)$



$+(r_i, r_j)$

Linear prefix form

Notation

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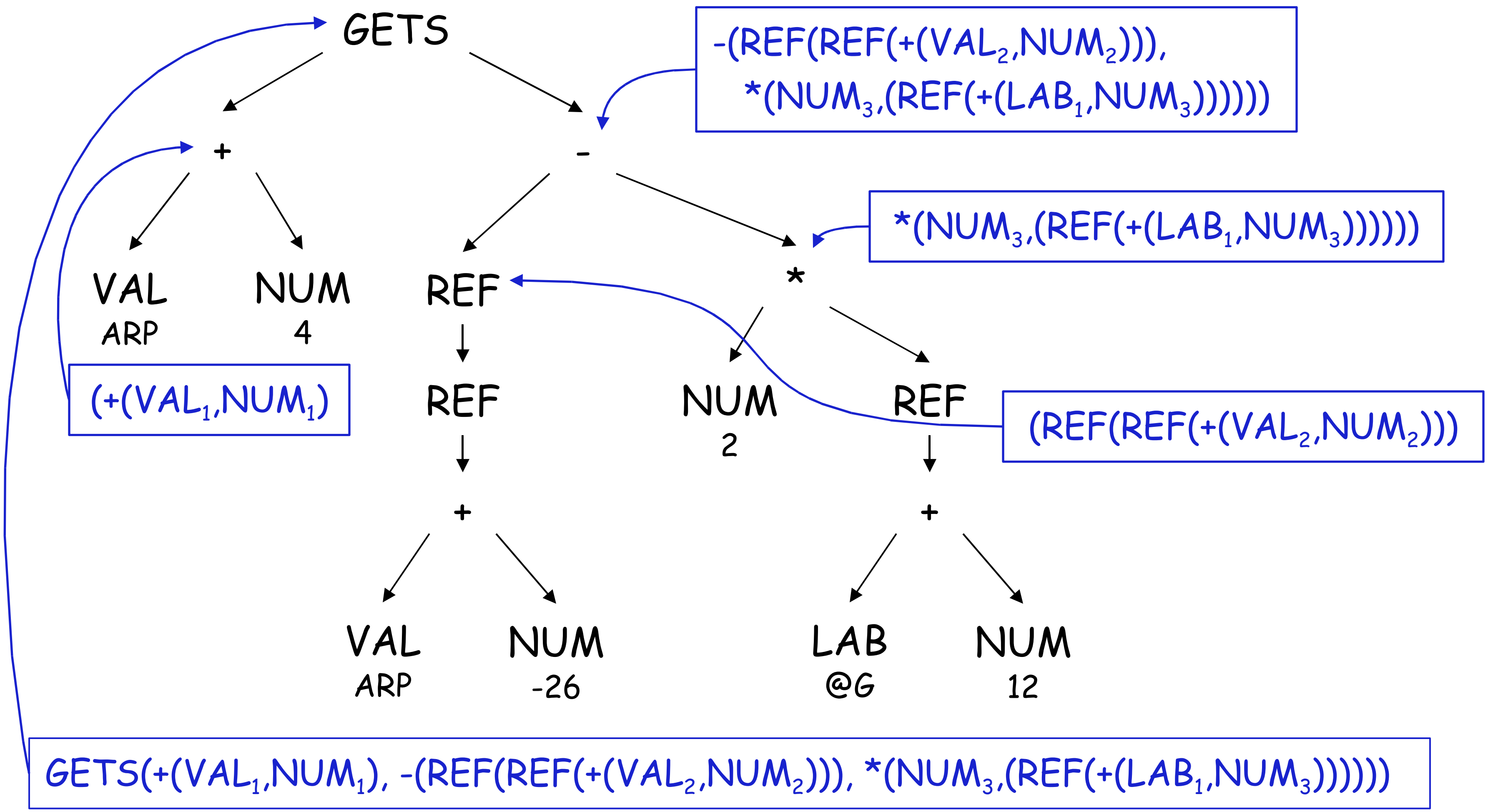


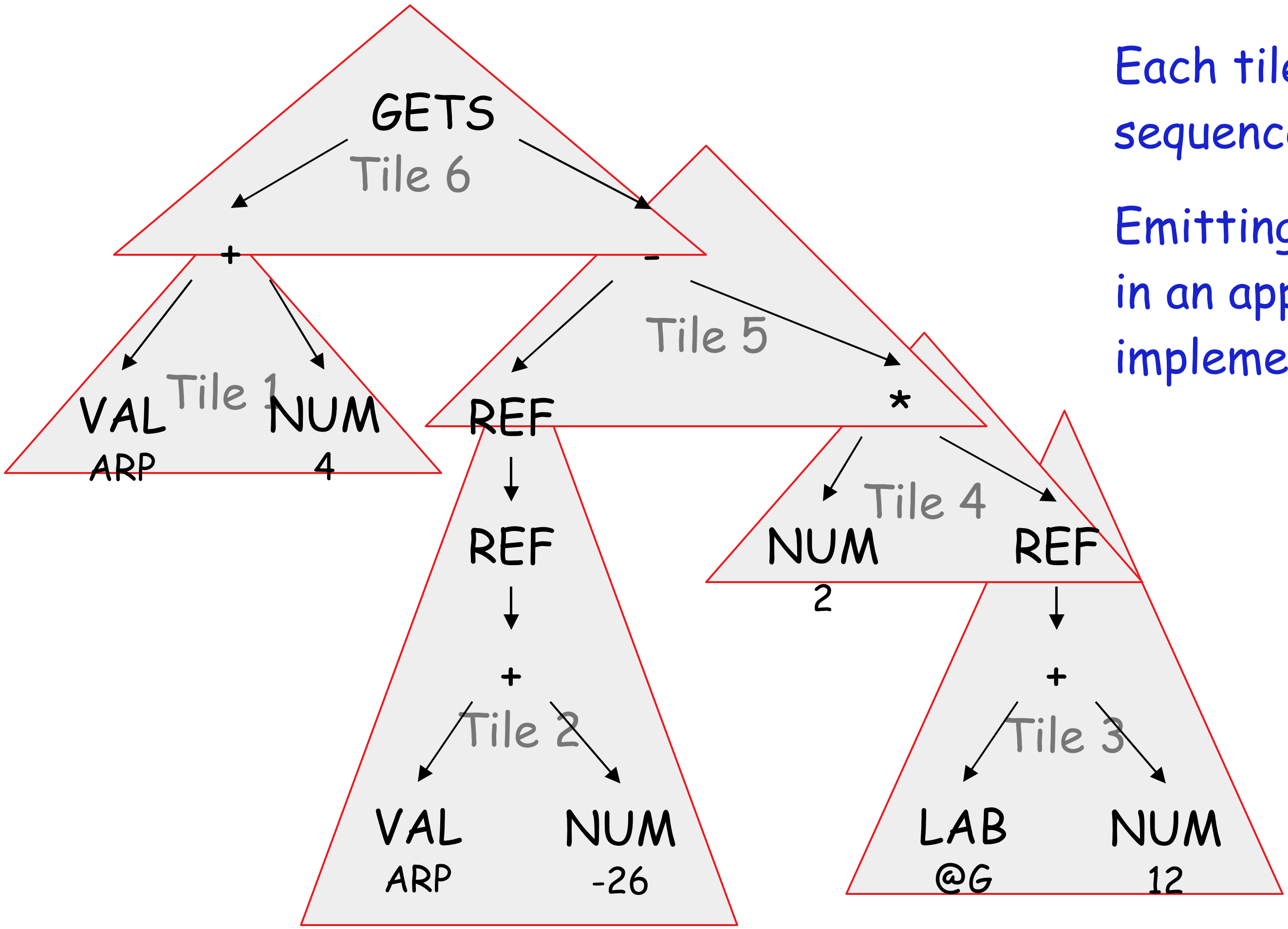
Tree-pattern matching

Goal is to "tile" AST with operation trees

- A tiling is collection of <*ast*,*op*> pairs
  - → *ast* is a node in the AST
  - → *op* is an operation tree
  - → <*ast*, *op*> means that *op* could implement the subtree at *ast*

- A tiling 'implements" an AST if it covers every node in the AST and the overlap between any two trees is limited to a single node
  - → <*ast*, *op*> ∈ tiling means *ast* is also covered by a leaf in another operation tree in the tiling, unless it is the root
  - → Where two operation trees meet, they must be compatible (expect the value in the same location)

Tiling the Tree



Each tile corresponds to a sequence of operations

Emitting those operations in an appropriate order implements the tree.

Generating Code

Given a tiled tree

- Postorder treewalk, with node-dependent order for children
  - → Right child of GETS before its left child
  - → Might impose "most demanding first" rule … *(Sethi)*
- Emit code sequence for tiles, in order
- Tie boundaries together with register names
  - → Tile 6 uses registers produced by tiles 1 & 5
  - → Tile 6 emits "`store` $\texttt{r}_{\texttt{tile 5}} \Rightarrow \texttt{r}_{\texttt{tile 1}}$"
  - → Can incorporate a "real" allocator or can use "NextRegister++"

So, What's Hard About This?

Finding the matches to tile the tree

- Compiler writer connects operation trees to AST subtrees
  - → Provides a set of rewrite rules
  - → Encode tree syntax, in linear form
  - → Associated with each is a code template

Rewrite rules: LL Integer AST into ILOC

	Rule	Cost	Template
1	Goal $\rightarrow$ Assign	0	
2	Assign $\rightarrow$ GETS(Reg_1,Reg_2)	1	store $r_2 \Rightarrow r_1$
3	Assign $\rightarrow$ GETS(+(Reg_1,Reg_2),Reg_3)	1	storeAO $r_3 \Rightarrow r_1, r_2$
4	Assign $\rightarrow$ GETS(+(Reg_1,NUM_2),Reg_3)	1	storeAI $r_3 \Rightarrow r_1, n_2$
5	Assign $\rightarrow$ GETS(+(NUM_1,Reg_2),Reg_3)	1	storeAI $r_3 \Rightarrow r_2, n_1$
6	Reg $\rightarrow$ LAB_1	1	loadI $l_1 \Rightarrow r_{new}$
7	Reg $\rightarrow$ VAL_1	0	
8	Reg $\rightarrow$ NUM_1	1	loadI $n_1 \Rightarrow r_{new}$
9	Reg $\rightarrow$ REF(Reg_1)	1	load $r_1 \Rightarrow r_{new}$
10	Reg $\rightarrow$ REF(+ (Reg_1,Reg_2))	1	loadAO $r_1, r_2 \Rightarrow r_{new}$
11	Reg $\rightarrow$ REF(+ (Reg_1,NUM_2))	1	loadAI $r_1, n_2 \Rightarrow r_{new}$
12	Reg $\rightarrow$ REF(+ (NUM_1,Reg_2))	1	loadAI $r_2, n_1 \Rightarrow r_{new}$

Rewrite rules: LL Integer AST into ILOC (*part II*)

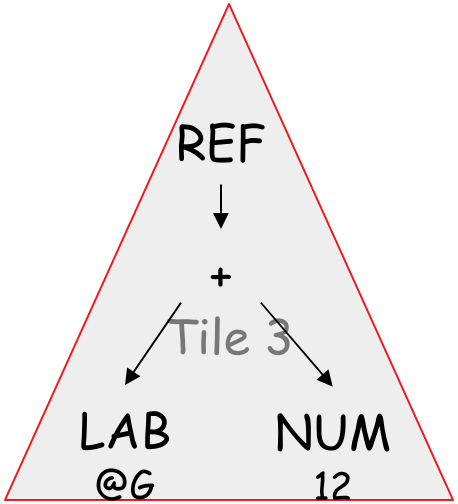
	Rule	Cost	Template
13	$Reg \rightarrow +\ (Reg_1, Reg_2)$	1	add $r_1, r_2 \Rightarrow r_{new}$
14	$Reg \rightarrow +\ (Reg_1, NUM_2)$	1	addI $r_1, n_2 \Rightarrow r_{new}$
15	$Reg \rightarrow +\ (NUM_1, Reg_2)$	1	addI $r_2, n_1 \Rightarrow r_{new}$
16	$Reg \rightarrow -\ (Reg_1, Reg_2)$	1	sub $r_1, r_2 \Rightarrow r_{new}$
17	$Reg \rightarrow -\ (Reg_1, NUM_2)$	1	subI $r_1, n_2 \Rightarrow r_{new}$
18	$Reg \rightarrow -\ (NUM_1, Reg_2)$	1	rsubI $r_2, n_1 \Rightarrow r_{new}$
19	$Reg \rightarrow x\ (Reg_1, Reg_2)$	1	mult $r_1, r_2 \Rightarrow r_{new}$
20	$Reg \rightarrow x\ (Reg_1, NUM_2)$	1	multI $r_1, n_2 \Rightarrow r_{new}$
21	$Reg \rightarrow x\ (NUM_1, Reg_2)$	1	multI $r_2, n_1 \Rightarrow r_{new}$

A real set of rules would cover more than signed integers …

So, What's Hard About This?

Need an algorithm to AST subtrees with the rules

Consider tile 3 in our example



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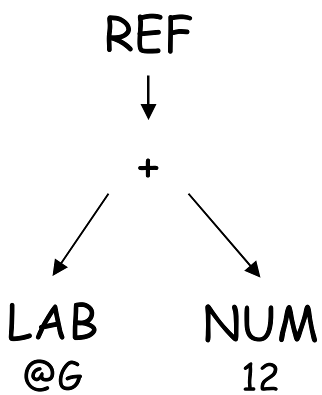
What rules match tile 3?



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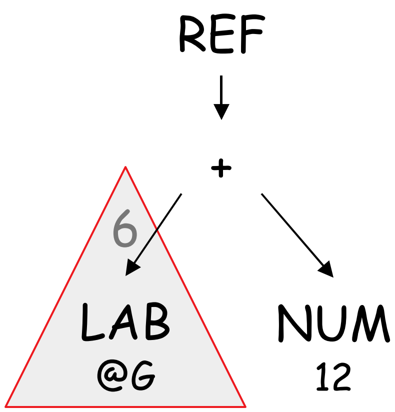

What rules match tile 3?

6: Reg $\rightarrow$ LAB_1 tiles the lower left node

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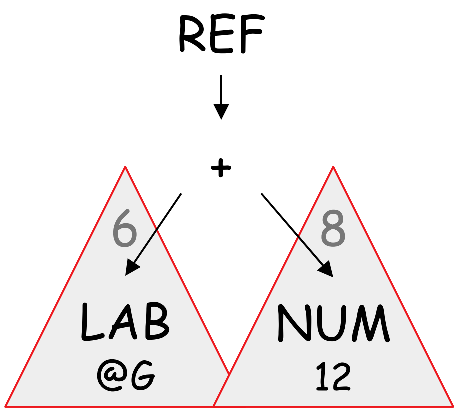

What rules match tile 3?

6: Reg $\rightarrow$ LAB_1 tiles the lower left node

8: Reg $\rightarrow$ NUM_1 tiles the bottom right node

So, What's Hard About This?

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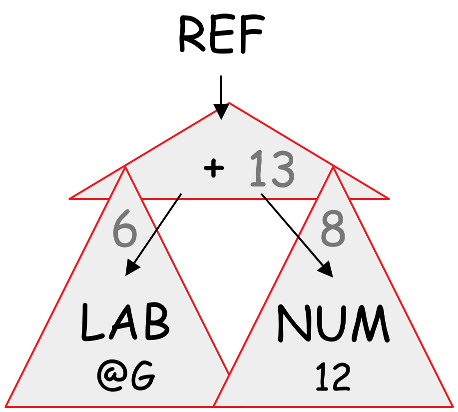
Consider tile 3 in our example



What rules match tile 3?

6: Reg $\rightarrow$ LAB_1 tiles the lower left node

8: Reg $\rightarrow$ NUM_1 tiles the bottom right node

13: Reg $\rightarrow$ + (Reg_1,Reg_2) tiles the + node

So, What's Hard About This?

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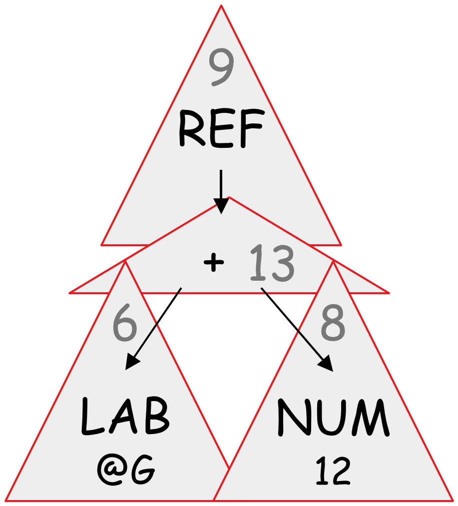
Consider tile 3 in our example



What rules match tile 3?

6: $\text{Reg} \rightarrow \text{LAB}_1$ tiles the lower left node

8: $\text{Reg} \rightarrow \text{NUM}_1$ tiles the bottom right node

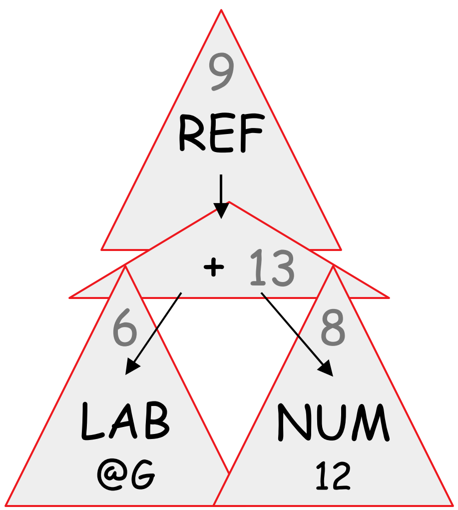
13: $\text{Reg} \rightarrow + (\text{Reg}_1,\text{Reg}_2)$ tiles the + node

9: $\text{Reg} \rightarrow \text{REF}(\text{Reg}_1)$ tiles the REF

So, What's Hard About This?

Need an algorithm to AST subtrees with the rules

Consider tile 3 in our example



What rules match tile 3?

6: $Reg \rightarrow LAB_1$ tiles the lower left node

8: $Reg \rightarrow NUM_1$ tiles the bottom right node

13: $Reg \rightarrow + (Reg_1,Reg_2)$ tiles the + node

9: $Reg \rightarrow REF(Reg_1)$ tiles the REF

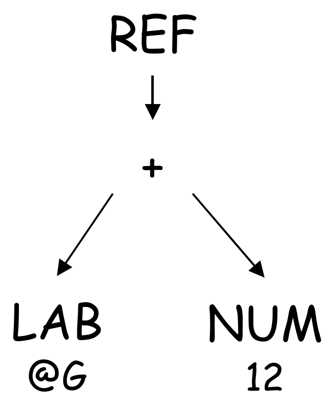
We denote this match as <6,8,13,9>

Of course, it implies <8,6,13,9>

Both have a cost of 4

Finding matches

Many Sequences Match Our Subtree



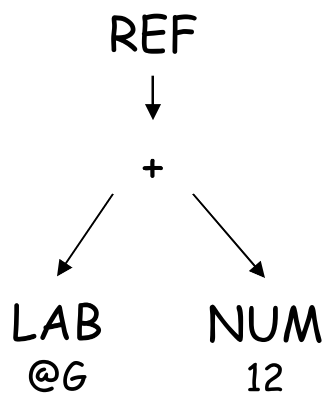
Cost	Sequences			
2	6,11	8,12		
3	6,8,10	8,6,10	6,14,9	8,15,9
4	6,8,13,9	8,6,13,9		

In general, we want the low cost sequence

- Each unit of cost is an operation (1 cycle)
- We should favor short sequences

Finding matches

Low Cost Matches



Sequences with Cost of 2	
6: $Reg \rightarrow LAB_1$ 11: $Reg \rightarrow REF(+(Reg_1,NUM_2))$	`loadI  @G   ⇒ r`$_i$ `loadAI r`$_i$`,12 ⇒ r`$_j$
8: $Reg \rightarrow NUM_1$ 12: $Reg \rightarrow REF(+(NUM_1,Reg_2))$	`loadI 12    ⇒ r`$_i$ `loadAI r`$_i$`,@G ⇒ r`$_j$

These two are equivalent in cost

6,11 might be better, because @G may be longer than the immediate field

Tiling the Tree

Still need an algorithm

- Assume each rule implements one operator
- Assume operator takes 0, 1, or 2 operands

Now, …

Tiling the Tree

```
Tile(n)
  Label(n) ← Ø
  if n has two children then
      Tile (left child of n)
      Tile (right child of n)
      for each rule r that implements n
         if (left(r) ∈ Label(left(n)) and
            (right(r) ∈ Label(right(n))
           then Label(n) ← Label(n) ∪ {r}
  else if n has one child
      Tile(child of n)
      for each rule r that implements n
         if (left(r) ∈ Label(child(n))
            then Label(n) ← Label(n) ∪ {r}
  else  /* n is a leaf */
     Label(n) ← {all rules that implement n}
```

Match binary nodes against binary rules

Match unary nodes against unary rules

Handle leaves with lookup in rule table

Tiling the Tree

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  else if n has one child
      Tile(child of n)
      for each rule r that implements n
          if (left(r) ∈ Label(child(n))
             then Label(n) ← Label(n) ∪ {r}
  else  /* n is a leaf */
      Label(n) ← {all  rules that implement n}
```

This algorithm

- Finds all matches in rule set
- Labels node n with that set
- Can keep lowest cost match at each point
- Leads to a notion of local optimality — lowest cost at each point
- Spends its time in the two matching loops

Tiling the Tree

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Oversimplifications

2. Only handles 1 storage class
3. Must track low cost sequence in each class
4. Must choose lowest cost for subtree, across all classes

The extensions to handle these complications are pretty straightforward.

Tiling the Tree

```
Tile(n)
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  else  /* n is a leaf */
      Label(n) ← {all rules that implement n}
```

Can turn matching code (inner loop) into a table lookup

Table can get huge and sparse

|op trees| x |labels| x |labels|
200 x 1000 x 1000
leads to 200,000,000 entries

Fortunately, they are quite sparse & have reasonable encodings (e.g., Chase's work)

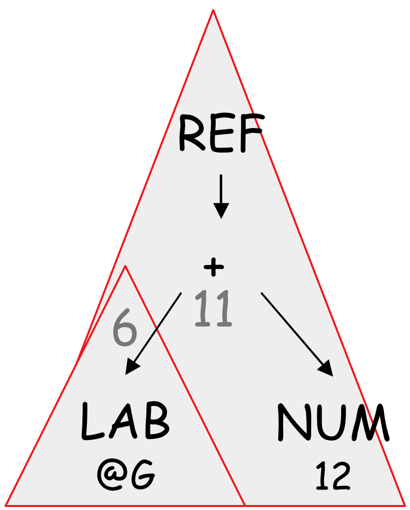
The Big Picture

- Tree patterns represent AST
- Can use matching algorithms to find low-cost tiling of AST
- Can turn a tiling into code using templates for matched rules
- Techniques (& tools) exist to do this efficiently

Hand-coded matcher like *Tile*	Avoids large sparse table Lots of work
Encode matching as an automaton	O(1) cost per node Tools like BURS, BURG
Use parsing techniques	Uses known technology Very ambiguous grammars
Linearize tree into string and use Aho-Corasick	Finds all matches

Extra Slides Start Here

Other Sequences



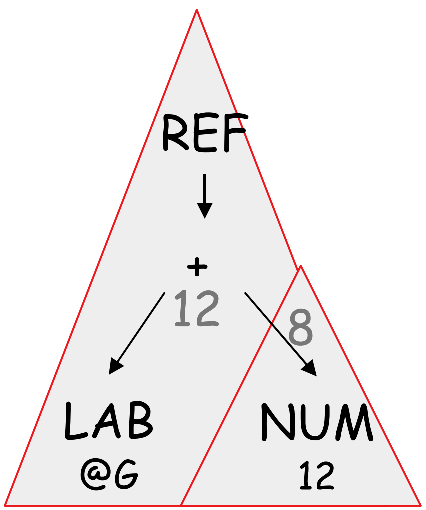
6,11

6: $Reg \rightarrow LAB_1$

11: $Reg \rightarrow REF(\ +\ (Reg_1, NUM_2))$

$\boxed{\text{Two operator rule}}$

Other Sequences



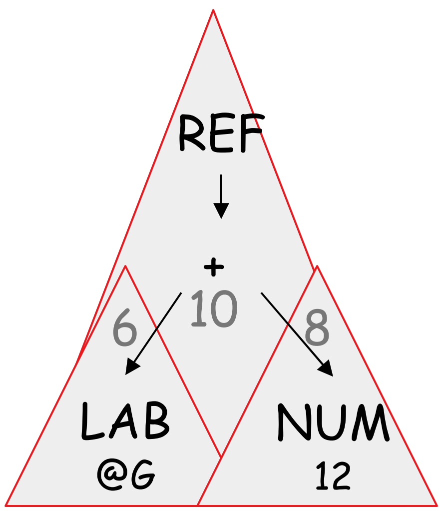
8,12

8: $Reg \rightarrow NUM_1$

12: $Reg \rightarrow REF(+(NUM_1, Reg_2))$

Two operator rule

Other Sequences



6,8,10

6: $Reg \rightarrow LAB_1$

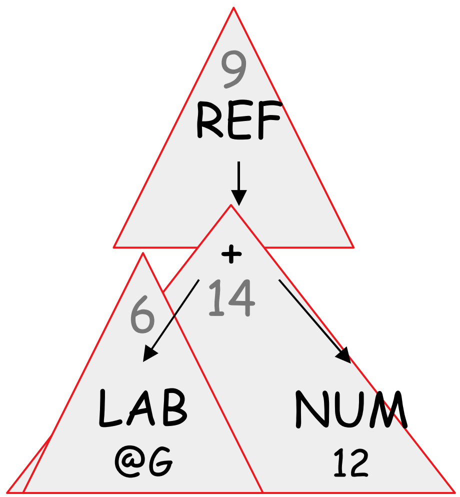
8: $Reg \rightarrow NUM_1$

11: $Reg \rightarrow REF(+ (Reg_1,Reg_2))$

Two operator rule

8,6,10 looks the same

Other Sequences



6,14,9

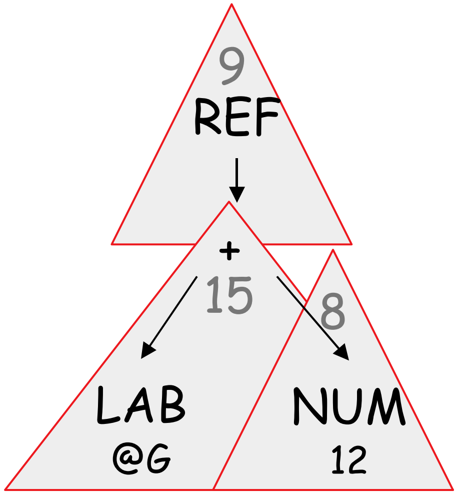
6: $Reg \rightarrow LAB_1$

14: $Reg \rightarrow + (Reg_1, NUM_2)$

9: $Reg \rightarrow REF(Reg_1)$

All single operator rules

Other Sequences



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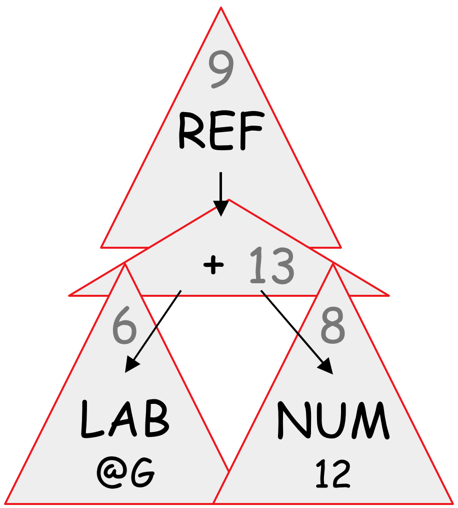

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